

Mathematics Program  
Transylvania University  
Lexington, KY 40508  
January 11, 2017



To the Members of the Bingham Renewal Selection Committee:

It is my pleasure to submit my application and supporting materials to be considered for the renewal of my Bingham award. This document contains a table of contents, my current curriculum vitae, my renewal narrative and pedagogical and scholarly supporting materials, including several recent syllabi. Please let me know if the committee requires any additional information. I look forward to meeting with the committee later this semester.

Sincerely,

A handwritten signature in black ink, which appears to read "Ryan Stufflebeam". The signature is fluid and cursive, with the first and last names being clearly legible.

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Mathematics Program Director  
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**Ryan Stufflebeam**  
**Bingham Award Renewal Application**  
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# CURRICULUM VITAE

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## EDUCATION

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- Ph.D. Mathematics**, 2004  
University of Iowa, Iowa City, Iowa  
Dissertation: *The Theta Correspondences for  $U(1)$  and the Quasi-split  $U(2)$*   
Advisor: Dr. David Manderscheid
- B.A. Mathematics and Computer Science**, *summa cum laude*, 1998  
Knox College, Galesburg, IL  
Honors in Mathematics  
Honors Thesis: *Distributions, Fourier Transforms and Fundamental Solutions*

## ACADEMIC POSITIONS

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**Transylvania University**, 2007 -

*Associate Professor and Program Director, Mathematics Program*  
Courses Taught: *Introduction to Contemporary Mathematics, Elementary Statistics, Calculus I, II and III, Linear Algebra, Differential Equations, The Mathematician's Toolkit, Number Theory, Modern Geometry, Abstract Algebra, Higher Analysis, Senior Seminar, Independent Studies in Abstract Algebra II, Topology and Complex Analysis, First-Year Research Seminar, Further Engagements, August Term*

**Ohio State University**, 2005 - 2007

*VIGRE Ross Assistant Professor, Department of Mathematics*  
Courses Taught: *Multivariable Calculus, Differential Equations, Lie Groups, Lie Algebras, Algebraic Groups*

**Boston College**, 2004 - 2005

*Visiting Assistant Professor, Department of Mathematics*  
Courses Taught: *Calculus I and II, Linear Algebra*

**University of Iowa**, 1999 - 2004

*Graduate Teaching Assistant, Department of Mathematics*  
Courses Taught: *Algebra II, Calculus II, Mathematics for Elementary School Teachers*

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## RESEARCH INTERESTS

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### Representation Theory of $p$ -adic Groups

- Construction of supercuspidal representations of low-dimensional linear groups
- Determination of eigenvectors of supercuspidals in the Weil representation
- Pairing of supercuspidals via local Howe duality

### Number Theory

- Weird fractions and their enumeration
- Factorizations of repunits
- Composite sequences and coverings of integers
- $2$ -adic integers and the Collatz Conjecture
- Transcendental and algebraic numbers
- Continued fractions, nested radicals and the characterization of numbers

### Commutative Ring Theory

- Local Noetherian Rings
- Indecomposable Modules and Decomposition Theory
- Monoids and the Factorization of Modules

### Pedagogy

- The use of writing in the mathematics curriculum

## SCHOLARSHIP and PROFESSIONAL ACTIVITY

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### Publications, Preprints and Works in Progress

- *How Weird are Weird Fractions?* **College Mathematics Journal**, **44** no.3, 202-209
- *Editorial for Special Issue on Writing and Editing in the Mathematics Curriculum: Part I* **PRIMUS**, **24** no. 6, 443-446 with Martin Montgomery
- *Editorial for Special Issue on Writing and Editing in the Mathematics Curriculum: Part II* **PRIMUS**, **24** no. 7, 557-559 with Martin Montgomery
- *The Composite Two-Step*. Co-author Eric Kaper, submitted
- *Not Too Big, Not Too Small: 'Goldilocks' Solutions to Certain Linear Diophantine Equations*. In preparation
- *On the Supercuspidal Representations of  $U(1,1)$* . Preprint
- *The Theta Correspondences for  $U(1)$  and the Quasi-split  $U(2)$* . Ph.D. Dissertation, Department of Mathematics, University of Iowa
- *Distributions, Fourier Transforms and Fundamental Solutions*. Honors Thesis, Knox College

### Editorial Work

- Associate Editor – *Special Issue on Writing and Editing in the Mathematics Curriculum: Part I* **PRIMUS**, **24** no. 6, 2014
- Associate Editor – *Special Issue on Writing and Editing in the Mathematics Curriculum: Part II* **PRIMUS**, **24** no. 7, 2014

### Session Organizer

- *Incorporating Writing and Editing into Mathematics Courses* – Contributed Paper Session, 2012 MathFest, Madison, WI
- *Novel Ways to Incorporate Writing into Mathematics Courses* – Contributed Paper Session, 2011 MathFest, Lexington, KY

## Presentations

- *The Composite Two-Step* – 2015 MathFest, Washington, D.C.
- *The Composite Two-Step* – 2015 KYMAA Annual Meeting, Morehead State University
- *Divisibility Desired: Coverings, Repunits and Composite Numbers* – Academic Affairs, Transylvania University, 2012
- *The Best of Both Worlds: Linear Diophantine Equations and Solutions of a Certain Size* – Contributed Paper Session on Open and Accessible Problems in Number Theory, 2012 MathFest, Madison, WI
- *How Weird are Weird Fractions?* – 2012 KYMAA Annual Meeting, Bellarmine University
- *Counting the Number of Correct Invalid Reductions of Fractions: How Weird are Weird Fractions?* – Contributed Paper Session, 2012 Joint Mathematics Meetings, Boston, MA
- *How Weird are Weird Fractions?* – 2012, Centre College
- *A Mistake by Any Other Name: The Curious Case of Weird Fractions* – Academic Affairs, Transylvania University, 2011
- *A “Novel” Approach to Writing in a Liberal Arts Math Class* – 2011 KYMAA Annual Meeting, Eastern Kentucky University
- *The Collatz Conjecture and the 2-adic Integers* – Contributed Paper Session, 2010 MathFest, Pittsburgh, PA
- *The Collatz Conjecture and the 2-adic Integers* – 2010 KYMAA Annual Meeting, University of Kentucky
- *Twin Hailstones and Parity: Two Views of the  $3n+1$  Problem* – Academic Affairs, Transylvania University, 2009
- *The Local Theta Correspondences for  $(U(1), U(1,1))$*  – Haar Seminar, Ohio State University, 2007
- *In Defense of James Gregory* – VIGRE Reading Classics Seminar, Ohio State University, 2006
- *On the Local Theta Correspondence for  $(U(1), U(1,1))$*  – Algebra Seminar, Boston University, 2005
- *On the Theta Correspondence for  $U(1)$  and the Quasi-split  $U(2)$*  – Special Session on the Representation Theory of Reductive Groups, AMS Sectional Meeting, Northwestern University, 2004
- *The Local Theta Correspondence for  $U(1)$  and the Quasi-split  $U(2)$*  – Number Theory Seminar, Boston College, 2004
- *Not Your Parents’ Absolute Value: Thinking  $p$ -adically* – Colloquium, Knox College, 2004

## UNDERGRADUATE RESEARCH SUPERVISION

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- Danielle Kane. “Estimates of the size of solutions to linear Diophantine equations and the representation of weird fractions.” 2012 - 2013
- Eric Kaper. “When does the continued concatenation of digits to a number result in a composite?” 2012
- Kelli Lang. “Twin hailstones and the  $3n+1$  algorithm.” 2009 - 2010

- Brittany Hulette and Karoline Weber. “Conjugacy classes of the general linear group of degree 2 over a finite field.” 2009

## PROFESSIONAL DEVELOPMENT

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- SACSCOC Annual Meeting. Atlanta, GA, 2016.
- Visualizing Projective Geometry workshop. Columbus, OH, MathFest 2016.
- SACSCOC Small College Initiative Workshop. Atlanta, GA, 2016.
- 2016 Kentucky Academic Leadership Academy.
- KYMAA Executive Board member and Student Chapters Coordinator. 2012 - 2015
- MAA PREP workshop on algebraic number theory. Williams College, 2010
- Inquiry-based learning workshop. Centre College, 2010
- “Beginning a Research Program in the Natural Sciences at a Predominantly Undergraduate Institution.” Council on Undergraduate Education. Davidson University, 2008
- Project NExT (New Experiences in Teaching). 2007 - 2008

## PROFESSIONAL SERVICE

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- *Mathematics Magazine* – Anonymous referee, 2016
- Served as anonymous external reviewer for a personnel decision at peer Kentucky institution, 2015.
- *PRIMUS* – Anonymous referee, 2013 - 2016.
- 2013 annual meeting of the Kentucky Section of the Mathematical Association of America (KYMAA) at Transylvania University April 5-6, 2013. Local coordinator
- KYMAA Executive Board member and Student Chapters Coordinator. 2012 - 2015
- “What Exactly is a Postdoc in Mathematics?” Panel member. University of Kentucky Graduate School, 2012
- Selection committee for KYMAA Distinguished College or University Teaching Award. Member, 2012
- Central Kentucky Math Circle facilitator, 2011 -
- Selection committee for KYMAA Officer Nominations. Member, 2011
- *Journal of Number Theory* – Anonymous referee, 2008 - 2010

## SERVICE TO THE UNIVERSITY

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|--|--------------|
| • SACS Certification Compliance Coordinator                      | 2016 -       |
| • Transylvania University New Faculty Roundtable, Co-Coordinator | 2015 -       |
| • Mathematics Search Committee, Chair                            | 2015 - 16    |
| • Transylvania University Branding Committee                     | 2015         |
| • Academic Eligibility Officer for the Athletics Department      | 2014 -       |
| • Presidential Transition Team                                   | 2014         |
| • Presidential Search Committee, Faculty Member                  | 2013 - 2014  |
| • Premier Scholarship Interviews                                 | 2012 - 2014  |
| • Judicial Board   | 2011 - 2012  |
| • Phi Beta Kappa Site Visit                                      | 2011         |
| • Mathematics Program Director                                   | 2010, 2012 - |
| • Teacher Education Advisory Board                               | 2010 -       |

• Grants Allocation Committee	2009 - 2011
• Board of Trustees Physical Plant Committee	2009 - 2011
• CPC Subcommittee on Diversity	2009 - 2012
• William T. Young Scholarship Committee	2009
• Barry M. Goldwater Scholarship Advisor	2008 -
• Math Club Advisor	2008 -
• Admissions Events	2007 -
• Mathematics Program's Problem of the Week	2007 -
• Mathematics Exam Competitions Coordinator	2007 -

## GRANTS, FELLOWSHIPS and AWARDS

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• Jones faculty development grant	2016
• Kenan grant to Danielle Kane, faculty mentor	2012
• Bingham Fellow	2011 -
• Knox College Athletic Hall of Fame	2011
• Kenan grant to Kelli Lang, faculty mentor	2009
• Kenan grant to Brittany Hulette and Karoline Weber, faculty mentor	2009
• Wethersfield High School (Kewanee, IL) Academic Hall of Fame	2009
• Bingham new faculty grant	2007
• Project NExT Exxon Mobil Fellow	2007 - 2008
• VIGRE Postdoc, Ohio State University	2005 - 2007
• University of Iowa Outstanding Teaching Assistant Award	2000 - 2001
• University of Iowa Presidential Fellow	1998 - 2004
• Knox College Faculty Scholar	1997 - 1998
• Knox College Lincoln Scholar	1994 - 1998

## PROFESSIONAL and HONORARY SOCIETIES

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Mathematical Association of America (MAA), Kentucky Section of the MAA (KYMAA), Phi Beta Kappa, Holleian Society

## **Ryan Stuffelbeam - Bingham Renewal Narrative**

During the past decade at Transylvania, my approach to pedagogy has evolved and continues to do so. The objectives discussed in my original Bingham Award application still hold true - creating an open and inviting learning environment and enabling students to interact with mathematics in a meaningful manner - but the manner in which I try to achieve these goals is never static. Since 2011, my teaching style has developed in numerous ways. Changes in how I use active learning and student assessment are most notable.

Due to a combination of training and course content coverage anxiety, mathematicians tend to use class time to deliver lectures. However, mathematics is a hands-on discipline and students need time to grapple with the material themselves. Ceding significant allotments of class time to student involvement, through activities such as working problems, presenting at the board, or group work, is the biggest - and most beneficial - change I have made as an instructor. This approach allows more frequent and meaningful interaction with each student and places a focus on each student's ability to work with the concepts at hand. Moreover, I am able to give students real-time feedback and clear up misconceptions before they take root.

My roles as mathematics program director and accreditation compliance certification coordinator have given me an appreciation of how effective student assessment can benefit student learning and instructor improvement. I am more intentional about aligning the particulars of any assignment with the student learning outcomes for the course. Furthermore, I want the assessment of graded material to be meaningful to both the students and myself. Finally, I want the assessment to reflect a given student's own understanding of the material. In this vein, I am beginning to replace graded homework with daily quizzes in most classes. In my opinion, quizzes provide a direct and purposeful means of assessment for everyone involved and, in this day of online algebra engines and solution sets, guarantee an accurate reflection of student learning.

### **Scholarship, Professional Activity, and Upper-Level Courses**

Over the past six years, my professional activity and pedagogy have been intertwined in several ways. Positive feedback from students and colleagues on my creative writing assignments encouraged me to organize and chair contributed paper sessions on writing and editing in mathematics courses at the 2011 and 2012 MathFests, the annual meeting of the Mathematical Association of America (MAA). This presented the opportunity to serve as guest editor for two special issues of PRIMUS, a peer-reviewed journal dedicated to mathematics pedagogy. In addition to my editorial duties, I co-authored the introduction to each special issue.

My involvement in the MAA has allowed me to stay current with pedagogical best practices. My work during an MAA-sponsored workshop in 2010 allowed me to spearhead a change to our program's capstone course, *Senior Seminar*. Beginning in 2011, we have run the course as a true seminar with students reading and presenting peer-reviewed papers. This change allows our majors more insight into the process of doing research mathematics and gives them a variation on the standard class model. Eric Kaper, a 2013 graduate, and I have a paper under review that answers a question posed in a paper used in a *Senior Seminar* presentation. I have participated in workshops on geometry and number theory pedagogy and presented classroom-related talks at MAA events. Also, my three years serving on the Executive Board of KYMAA (MAA's Kentucky section) as student chapters coordinator helped me build a network of Kentucky-based colleagues with whom I can compare notes and trade teaching ideas.

My upper-level courses also benefit from appropriate uses of technology. In my *Modern Geometry* course, I use online applets detailing a model of hyperbolic geometry to give students tangible experience with non-Euclidean geometry. In my *Number Theory* course, students utilize the power of



modern computational engines to encrypt and decrypt messages to get hands-on experience with cryptography; this process was not computationally feasible when I was a student. Also, I use a computer algebra system in *Linear Algebra* and *Calculus III* to enable students to better visualize mathematical objects in 3-dimensional space. These choices are made with improving the student learning experience in mind.

Finally, while the bulk of the math program's annual assessment concerns general education courses, we have begun to better use this tool for our upper-level courses. This is a needed and important adjustment; I believe it is necessary to have factual evidence that our students are learning effectively in our upper-level courses and that our major is rigorous. I have attached the portion of the program's 2015-16 annual assessment detailing an outcome associated with my offering of *The Mathematician's Toolkit* in May 2016.

## **The Calculus Sequence**

Mathematics departments everywhere seek to decrease the number of grades of D or below and course withdrawals in precalculus and calculus courses; our program is no exception. As program director, I led a study of student performance in *Functions*, our precalculus course offered each fall, and *Calculus I*. As a result, our program raised the prerequisite ACT-Math score for *Calculus I* to 26 from the previous 23 and switched from offering one section of *Functions* to two beginning in the fall of 2013. I used this as an opportunity to adjust my approach to these courses.

*Functions* is a skills-acquisition course for students; it does not count towards Transylvania's general education requirement and is taken by students who need *Calculus I* but do not meet its prerequisite. Since 2013, I have employed a flipped classroom in each of my three *Functions* sections. I record video lectures using an iPad app and students are required to watch the video and attempt homework problems before the next class session. During class, students continue working on problems and, if common questions arise, we work problems at the board together. Each class session ends with a quiz over the related material. Based on their feedback, students appreciate the frequent individual interaction this style allows and seem more prepared to complete *Calculus I*. As part of the mathematics program's 2016-17 self-assessment, I will conduct a four-year retrospective study to gauge the effectiveness of the changes made in 2013-14. (Videos are password-protected on [educreations.com](http://educreations.com) and are available upon request.)

While the content in the Calculus sequence remains consistent, the wide-ranging mathematical backgrounds of our students make teaching these courses a continual challenge. This diversity in student mathematical ability is felt acutely in *Calculus I*. Though our curricular change ensures any student taking *Calculus I* meets a more rigorous prerequisite, the range in student mathematical aptitudes remains widespread. The variance in problem-solving techniques - from applying high-level calculus concepts to computing a derivative via basic algebra - requires students to be nimble and draw upon the entirety of their mathematical knowledge. To aid students, I distribute daily supplements - containing relevant theorems, techniques, and definitions - to serve as signposts for the course. When solving problems in class, I make sure to differentiate between new techniques and those previously encountered; in this way, I hope students begin to see the gradual accumulation of their calculus knowledge and of their arsenal of problem-solving techniques.

The material in our calculus sequence is consistent with our peer institutions. However, Transylvania's 13-week semesters make covering all necessary content in a substantial manner a challenge. This time crunch makes finding the appropriate balance between lecture and student activity during class my biggest obstacle as an instructor. Early in my career, I was hesitant to transfer control of class time to students but I have learned to place trust in Transylvania students. I have been rewarded by their ability to motivate themselves to learn new topics, especially in *Calculus II* and *III*. As previously noted, I am changing my grading methods and plan to make daily quizzes the

primary non-exam mode of assessment in each of the calculus courses. Ultimately, I believe this decision will benefit student learning but I recognize it will add to the perceived time pressure.

## General Education Courses

The Transylvania mathematics program offers two general education courses: *Introduction to Contemporary Mathematics* and *Elementary Statistics*. For the majority of students in *Contemporary* or *Statistics*, this is their only college mathematics course and many are hesitant to take such a course. Couple this with the diversity of enrolled students - in terms of mathematical background, educational maturity, and intellectual curiosity - and any instructor is faced with a significant task. Establishing our classroom as a welcoming space where every inquiry is welcomed and expected is my first duty in these courses. To further promote student learning, I distribute, via a shared Dropbox folder, lecture slides 24 hours in advance of the next class meeting. This gives students a sense of the day's topic and allows students to view the lecture in their desired format: on a tablet, a laptop, printed pages, or on the classroom screen.

Designing and maintaining a relevant *Statistics* course has been a major pedagogical challenge for me. My modest background in statistics forced me to put more time and thought into creating meaningful and useful lessons, examples, and assessment questions. I try to use practical real world applications of statistics in my exams; I have used auction prices from Keeneland horse sales and local weather data to construct exam problems. Also, I introduce students to user-friendly statistical applets and use this technology to reinforce concepts covered in class. Another perk of distributing lecture slides before class is the ability to do numerous examples without having to spend time writing the problem on the board. This allows the freedom to work on problems as a class or individually, giving me time to check in on each student's progress in real time, a necessary assessment in *Statistics* as the dependence on previous concepts increases as the semester advances.

In *Contemporary*, I continue to use new versions of the creative writing assignments mentioned earlier. By allowing students to be creative in the presentation of their solution, I hope students begin to view the field of mathematics as more than using equations to solve problems. Also, in *Contemporary*, I have already made the change to daily quizzes from graded homework and, thus far, have found the quizzes to be a more reliable and true indicator of a student's performance in the class.

## Moving Forward

I foresee several personal pedagogical challenges in the coming years. My top priority is to continue searching for the right balance between lecture and active student learning to provide our students with an effective in-class experience. Appropriate use of technology in the classroom is another issue I plan to reexamine. The mathematics program recently purchased a site license for the computer algebra system MATLAB and I am eager to become acquainted with this program and see how it can benefit our students, particularly in the calculus sequence. Also, I plan to stay updated on current technological trends through my involvement in the MAA, both local and national.

The educational experience of current incoming college students has dramatically changed from the past; the widespread use and ubiquity of tablets, online tutorials, educational apps, and video lessons is changing the learning habits and modes of delivery for today's student. Flipping the classroom in *Functions* was my first major foray into this new world and it has been a worthwhile endeavor. Over the next five years, I will continue to seek out novel pedagogical techniques to meet the ever-changing needs of our student body. Finally, I devoted my sabbatical and the year prior to beginning a new line of research and am ready to fully involve Transylvania undergraduates in this work. Ultimately, I hope to continue to learn and grow as a faculty member and provide our students every opportunity to succeed in reaching their potential.

## Supporting Materials - Teaching

As noted in the narrative, I believe that programmatic assessment can be better utilized by faculty members and programs to improve student instruction. As program director, I have been responsible for the mathematics program assessments since 2012. I have attached a page from our 2015-16 submission detailing the program's assessment of my May 2016 offering of *The Mathematician's Toolkit*.

Since the receipt of a Bingham Award in the spring of 2011, I have taught 12 different mathematics courses and led one independent study. I have included the most recent syllabi from eight of these courses: *Introduction to Contemporary Mathematics*, *Elementary Statistics*, *Functions*, *Calculus I*, *Calculus III*, *The Mathematician's Toolkit*, *Abstract Algebra*, and *Higher Analysis*. I am happy to supply syllabi from more classes if necessary.

During my time at Transylvania, I have developed and incorporated a nonstandard writing component into a handful of my courses. This endeavor was motivated by my participation in several MAA workshops. I developed a series of mathematical stories starring the mathematical wunderkind Wikipedia Green. Each story involves a problem whose solution is to be solved by Wiki using mathematics. The students' task is to solve the problem and write a conclusion to the story detailing their results. A cover letter providing instructions accompanies the assignment. The assignment provided is used in the financial mathematics segment of my *Introduction to Contemporary Mathematics* class.

<p>Mathematics majors and minors will gain facility in mathematical reasoning, recognizing and applying mathematical definitions and using proofs and counterexamples to determine whether a mathematical statement is true or false.</p>	<p>Exam questions in The Mathematician's Toolkit, the math program's introduction to higher mathematics and proofs.</p>	<p>Beginning in May of 2009, the math program has offered The Mathematician's Toolkit, an intensive 4-week course (during Transy's May Term) that introduces math majors and minors to the type of mathematics found in the upper portion of the mathematics curriculum.</p> <p>During May Term of 2016, 26 potential math majors and minors enrolled in the course. The following data comes from 3 questions on the exam administered at the end of the 4-week term.</p> <p>The questions were chosen to represent the different types of knowledge addressed in the course: <b>Question 1</b> asked students to use mathematical definitions to precisely construct a specific mathematical object; <b>Question 2</b> asked students to prove a statement via proof by induction, a basic method of proof in the upper-level curriculum; <b>Question 3</b> presented students with 2 statements - the students were asked to determine which was true and which false, to give a proof of the valid statement and a counterexample to disprove the false one. The average score out of 10 was:</p> <p><b>Question 1</b> - 8.33  <b>Question 2</b> - 9.35  <b>Question 3</b> - 7.08</p>	<p>The Mathematician's Toolkit serves as the foundation for majority of the courses in the mathematics major. As such, the program hopes that students gain facility with the new modes of thought developed during the course. Question 2 dealt with proof by induction and, as such, the students should have known how to do the question given its phrasing; the 9.35 average score indicates that students gained facility with this proof technique.</p> <p>Question 1 made students take an abstract definition and use it to construct a concrete set of numbers. The 8.33 average score indicates that some students were still learning how to go from the abstract to the concrete.</p> <p>Question 3 is more difficult than the other two because it asked students to classify statements as true or false and justify why they made these classifications. The average score of 7.08 (out of 10) indicates the uneasiness of some students in being able to determine true/false values without guidance.</p> <p>As questions 1 and 3 were more ambiguous in how a student could approach an answer, the math program will use this data in the Toolkit course to ask more questions that force students to identify statements as true/false and to apply concepts/definitions/theorems in tangible settings.</p>
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# MATH 1034-02 Introduction to Contemporary Mathematics

## Winter 2017

### Syllabus

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Dr. Ryan Stufflebeam  
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BSC 119            233-8237

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**Overview**    The main objective of this course is to provide an introduction to real-world applications of mathematical ideas and thinking. As described in the Transy Course Catalog, one goal “is to develop conceptual understanding and appreciation, not necessarily computational expertise.” In other words, we’ll be studying practical everyday uses of mathematics. As an Area I course, discovering how mathematics interacts with other branches of knowledge is another objective. Over the course of the semester, the following topics will be covered (with associated fields of study in parentheses): distributions and statistics (economics, social sciences, natural sciences), probability models (economics, social sciences, natural sciences), savings and borrowing models (business, accounting, economics, life in general!) and voting theory (law, political science, economics, business). The only mathematical knowledge assumed is basic arithmetic and elementary algebra; your willingness to work will have the biggest impact on your success in the class.

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**Student Learning Outcomes**    After successful completion of Introduction to Contemporary Mathematics, a Transylvania student will be able to:

- Analyze and solve problems involving basic statistical principles
- Understand and solve problems dealing with discrete probability models
- Work with equations detailing basic financial models
- Analyze problems in basic voting theory and be able to determine winners in elections using weighted voting systems

Throughout the term, a student’s capability in each of these areas is assessed through exams, class participation, writing assignments, and daily quizzes.

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**Class**    Class meets every Monday, Wednesday and Friday from 11:30 to 12:20 in BSC 108. Your attendance is expected at each meeting. If you must miss class, please inform Dr. Stufflebeam as soon as possible *before* your absence. Each unexcused absence beyond the third will result in a lowering of one’s course grade by a full plus/minus letter grade. Arriving to class more than 10 minutes late will be considered an unexcused absence. Most class periods will involve 35-40 minutes of lecture followed by a 10-15 minute quiz.

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**Text**    The required text for the course is *For All Practical Purposes: Mathematical Literacy in Today’s World*, Tenth Edition, by the COMAP Consortium. Dropbox will be used to distribute the set of lecture notes.

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**Course Grading** The following percentages will be used in determining your final course grade:

Quizzes	—	15 %
Exams I - V	—	14 % each
Writing Assignments	—	15 %

In calculating the quiz portion of your grade, the lowest score will be dropped. During the course the following scale will be used; it may be expanded but will *not* be contracted: 100-93 A; 92-90 A-; 89-87 B+; 86-83 B; 82-80 B-; 79-77 C+; 76-73 C; 72-70 C-; 69-67 D+; 66-63 D; 62-60 D-; 59-0 F.

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**Exams** There will be **five** exams during the course of the term. Each exam will be given during the normal class period - save for Exam V which will be administered during the final exam period. The scheduled dates for these midterms are

Monday January 30,	Monday February 13,
Friday March 3,	Monday March 27, Tuesday April 18

If, for some reason, an exam needs to be rescheduled, notice will be given at least a week in advance. During pre-exam review sessions, the possible material to be covered on an exam will be discussed. Makeup exams will be given only in extraordinary circumstances. Notice that there is NO cumulative final exam; the fifth exam - covering the last three weeks' worth of material - will be administered during the time set aside for our class during Finals Week.

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**Quizzes and Homework** In order to learn mathematics, one must **do** mathematics. To help kickstart your learning, homework will be assigned during each lecture period with a quiz over that material at the end of the next lecture meeting. The quiz material will be directly related to the assigned homework (but won't be the exact same problems). You are encouraged to work with others on homework. However, quizzes will be solitary endeavors, so be sure to arrive at your own understanding of the covered material. Your participation in class is welcome and expected. Though not an explicitly-defined part of the grade, your participation may be used to adjust the final homework grade.

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**Writing Assignments** There will be three writing assignments during the term. The specifications and expectations for these assignments will be discussed in class. Basically, each writing assignment will involve solving a nontrivial problem and composing your solution to adhere to specified guidelines. The (extremely tentative) **due** dates for the writing assignments are Friday February 17, Wednesday March 22 and Friday April 14. Each writing assignment will be made approximately a week and a half prior to the due date.

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**Office Hours** If my door is open, you are welcome to stop in and ask questions. Additionally, you may set up an appointment for a meeting. During the following times, I am guaranteed to be in (or around) my office:

Monday	—	10 : 30 — 11 : 20, 12 : 30 — 2 : 20
Tuesday	—	9 : 30 — 11 : 20
Wednesday	—	10 : 30 — 11 : 20, 1 : 30 — 2 : 20
Friday	—	10 : 30 — 11 : 20

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**Academic Integrity** As detailed in the Transylvania University Course Catalog:

*“Honesty, trust, and personal responsibility are fundamental attributes of the University community. Academic dishonesty by a student will not be tolerated, as it threatens the foundation of an institution dedicated to the pursuit of knowledge. To maintain its credibility and reputation, and to equitably assign evaluations of scholastic and creative performance, Transylvania University is committed to maintaining a climate that upholds and values the highest standards of academic integrity.”*

It is expected that each of you will conduct your academic studies in a manner adhering to these guidelines. As mentioned above, you are encouraged to work with other students when studying and attempting homework. It is expected that each individual submission reflect its author’s understanding of the covered material. All exams are individual endeavors and should be treated as such.

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**Classroom Atmosphere** Your participation is an integral part of the course. You are reminded, however, that there are other students in the class and non-mathematical off-topic discussions should be kept to a bare minimum. Drinks (water, coffee, etc.) are allowed in class, but all food items should be finished *before* lecture begins! Cell phones and all other feats of technology (save for the possibility of laptops/tablets) are to be turned *OFF* during class. Anyone using a cell phone, be it for talking, texting or surfing the net, during class will be asked to stop. A second such occurrence will result in that student being asked to leave class. Upshot: don’t use your cell phones!!

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**Lather, Rinse, Repeat** The following is a set of recommendations to help you make your way successfully through the course. Mathematics is most definitely NOT a spectator sport; to perform to your potential and achieve success in the course, you must make the effort and take the time to do and understand the homework problems and lecture examples. It is strongly advised that you use the provided lecture schedule to read the material to be covered in class ahead of time; minimally, one should at least browse the material prior to lecture. In any case, it is **imperative** that you read the text; this text is very readable and was written to appeal to the non-mathematically-inclined.

Do not wait until the night before an assignment is due to begin it. As a rough estimate, one should expect to spend 2.5-3 hours studying outside of class for each class meeting; having your text open while watching *The Expanse* does not count! This class moves fast! I encourage you to engage in good study habits from the beginning of the course. Please take time to work on and think about each of the homework problems; many problems will require more than a minute’s thought, so be persistent! Do not be afraid to make use of the other students in class as well as my availability. If questions arise or problems occur, please do not hesitate to contact me. Finally, please let me know of any circumstances which may affect your performance in the course.

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**Tentative Course Schedule** The following is the tentative daily schedule for MATH 1034-02. Changes to this schedule will be announced in class.

Monday	Wednesday	Friday
1/9 9.1	1/11 9.2	1/13 9.2-9.3
1/16 NO CLASS	1/18 9.3	1/20 9.3-9.4
1/23 10.1-10.2	1/25 10.3	1/27 10.3
1/30 <b>EXAM I</b>	2/1 11.1	2/3 11.1
2/6 11.2	2/8 11.2	2/10 11.2
2/13 <b>EXAM II</b>	2/15 5.1-5.2	2/17 5.3
2/20 5.4	2/22 5.5-5.6	2/24 5.7
2/27 5.8	3/1 5.9	3/3 <b>EXAM III</b>
3/6 8.1	3/8 8.2-8.3	3/10 8.2-8.3
3/13 SPRI	3/15 NG B	3/17 REAK!
3/20 8.5	3/22 8.5	3/24 8.4
3/27 <b>EXAM IV</b>	3/29 21.2	3/31 21.2
4/3 21.3	4/5 21.4	4/7 21.4
4/10 22.2	4/12 22.3	4/14 22.3

The Fifth Exam is Tuesday April 18 from 12 to 2 PM in BSC 108. For your reference, the drop date for the winter semester is Thursday January 26 and the withdrawal date is Friday March 3.



# MATH 1144-02 Elementary Statistics

## WINTER 2016

### Syllabus

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Dr. Ryan Stuffelbeam  
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BSC 119                      233-8237

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**OVERVIEW** From newspapers to the 5 o'clock news, political advertisements to opinion polls, the use of statistics is commonplace in today's society. Unfortunately, so is its misuse. Our main goal in this course is to develop a solid understanding of basic statistics and probability and be able to distinguish a proper statistical argument from a misleading one. Along the way we will encounter many topics: random sampling, representations of data, numerical summaries of data, relationship between two sets of data, the binomial probability distribution, the normal probability distribution, the sample mean and sample proportion, confidence intervals and hypothesis tests. As an Area I course, a second goal is to achieve an understanding of the basic mathematical principles used in the course and become conversant in using statistical formulas. Assignments will often make use of Excel. Some of our work will be computationally heavy; it is advised that you have access to a calculator.

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**CLASS** This course meets Monday, Wednesday and Friday from 1:30 to 1:20 PM in CC 202. Attendance at each and every class meeting is expected. If it is necessary for you to miss a class, please inform Dr. Stuffelbeam as soon as possible *prior* to your absence. Each unexcused absence beyond the second will result in a lowering of one's course grade by a full plus/minus letter grade. Arriving to class more than 10 minutes late will be considered an unexcused absence.

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**TEXT** The required text for the course is Michael Sullivan's *Fundamentals of Statistics*, Fourth Edition. As an overwhelming majority of the assigned work in this course stems from this text, it is fundamental (ha!) that you have continual access to a copy.

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**COURSE GRADING** The following percentages will be used in determining your final course grade:

Homework	—	20 %
Exams I - III	—	20 % each
Final Exam	—	20 %

In calculating the homework portion of your grade, the lowest *two* homework scores will be dropped. During the course the following scale will be used; it may be expanded but will *not* be contracted: 100-93 A; 92-90 A-; 89-87 B+; 86-83 B; 82-80 B-; 79-77 C+; 76-73 C; 72-70 C-; 69-67 D+; 66-63 D; 62-60 D-; 59-0 F.

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**EXAMS** There will be three midterm examinations during the term. Each exam will consist of both an in-class and take-home portion. They are scheduled for

Wednesday February 3,      Friday February 26,      Friday April 1

The take-home portion will be distributed at the end of the in-class examination and will be due during one of the class periods immediately following the exam. If, for some reason, an exam needs to be rescheduled, notice will be given at least a week in advance. During pre-exam review sessions, the possible material to be covered on an exam will be discussed. Makeup exams will be given only in extraordinary circumstances. The Final Exam is cumulative and will be administered Thursday April 21 from 9:00 - 11:00 AM in CC 202.

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**HOMEWORK** In order to learn mathematics, one must **do** mathematics. In an attempt to motivate your learning, homework assignments will be made on a daily basis. Generally, a homework set assigned during one class will be due at the beginning of the next class meeting. **LATE** homework will **NOT** be accepted. You are encouraged to work with others on homework. However, the work you submit should accurately reflect *YOUR* understanding of that day's material (i.e. Don't Cheat!). A few guidelines for the submission of homework: use standard paper *not* torn from a spiral notebook, staple multiple pages, show all work and write legibly in a coherent, logical manner. Your participation in class is welcome and expected. Though not an explicitly-defined part of the grade, your participation may be used to adjust the final homework grade.

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**OFFICE HOURS** If my door is open, you are welcome to stop in and ask questions. Additionally, you may set up an appointment for a meeting. During the following times, I am guaranteed to be in (or around) my office:

Monday	–	8 : 30 – 9 : 20, 10 : 30 – 11 : 20, 2 : 30 – 3 : 20
Tuesday	–	9 : 30 – 11 : 20
Wednesday	–	8 : 30 – 9 : 20
Friday	–	8 : 30 – 9 : 20, 10 : 30 – 11 : 20

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**ACADEMIC INTEGRITY** As detailed in the Transylvania University Course Catalog:  
*“Honesty, trust, and personal responsibility are fundamental attributes of the University community. Academic dishonesty by a student will not be tolerated, as it threatens the foundation of an institution dedicated to the pursuit of knowledge. To maintain its credibility and reputation, and to equitably assign evaluations of scholastic and creative performance, Transylvania University is committed to maintaining a climate that upholds and values the highest standards of academic integrity.”*

It is expected that each of you will conduct your academic studies in a manner adhering to these guidelines. As mentioned above, you are encouraged to work with other students when studying and completing homework. It is expected that each individual submission reflect its author's understanding of the covered material. All exams are individual endeavors and should be treated as such.

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**CLASSROOM ATMOSPHERE** Your participation is an integral part of the course. You are reminded, however, that there are other students in the class and non-mathematical off-topic discussions should be kept to a bare minimum. Drinks (water, coffee, etc.) are allowed in class, but all food items should be finished *before* lecture begins! Cell phones, iPhones and all other feats of technology (save for the possibility of laptops or tablets - for the express purpose of taking notes) are to be turned *OFF* during class. Anyone using a cell phone, be it for talking, texting or surfing the net, during class will be asked to stop. A second such occurrence will result in that student being asked to leave class. Upshot: don't use your cell phones!!

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**LATHER, RINSE, REPEAT** The following is a set of recommendations to help you make your way successfully through the course. Mathematics is most definitely NOT a spectator sport; to perform to your potential and achieve success in the Elementary Statistics course, you must make the effort and take the time to do and understand the homework problems and lecture examples. It is strongly advised that you use the provided lecture schedule to read the material to be covered in class ahead of time; minimally, one should at least browse the material prior to lecture. Also, lecture notes will be posted to the designated Dropbox folder for the class (at least) by the night before the lecture. In any case, it is **imperative** that you read the text. Do not wait until the night before an assignment is due to begin it. As a rough estimate, one should expect to spend

2.5-3 hours studying outside of class for each class meeting (having your text open while watching *Modern Family* does not count!) Do not be afraid to make use of the other students in class as well as my availability. If questions arise or problems occur, please do not hesitate to contact me. Finally, please let me know of any circumstances which may affect your performance in the course.

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**TENTATIVE COURSE SCHEDULE** The following is the tentative daily schedule for MATH 1144-02. Changes to this schedule will be announced in class.

Monday	Wednesday	Friday
1/11 Intro - 1.1	1/13 1.2-1.3	1/15 1.5-1.6
1/18 NO CLASS	1/20 2.1	1/22 2.2
1/25 3.1	1/27 3.2	1/29 3.4
2/1 3.5	2/3 <b>EXAM I</b>	2/5 4.1
2/8 4.2	2/10 4.3	2/12 6.1
2/15 6.1-6.2	2/17 6.2	2/19 7.1
2/22 7.2	2/24 7.3	2/26 <b>EXAM II</b>
2/29 8.1	3/2 8.1-8.2	3/4 8.2
3/7 9.1	3/9 9.1-9.2	3/11 9.2
3/14 SPRI	3/16 NG BR	3/18 EAK!
3/21 10.1	3/23 10.2	3/25 10.2-10.3
3/28 10.3	3/30 10.4	4/1 <b>EXAM III</b>
4/4 11.1	4/6 11.1	4/8 11.2
4/11 11.2-11.3	4/13 11.3	4/15 Review

The Final Exam is Thursday April 21 from 9-11 AM in CC 202. For your reference, the drop date for the term is Tuesday January 26 and the withdrawal date for fall term is Thursday March 4.

# MATH 1214-01 Functions

## Fall 2016

### Syllabus

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**OVERVIEW**     The main objective of this course is to prepare students for Calculus I. After introducing the general concept of a function, the rest of our time is spent examining and characterizing various families of functions. For sake of comparison, one may think of the Calculus sequence as a play, full of plot twists and turns, stage directions and character descriptions, asides and monologues. MATH 1214 serves as an introduction to the characters involved in this play. Our goal is to become intimately familiar with the basic mathematical objects that play leading roles in the Calculus.

Subjects to be covered include: the general theory of functions, graphs of functions, polynomial and rational functions, exponential and logarithmic functions, the unit circle, angles and radians, trigonometric functions and the inverse trigonometric functions. Students successfully completing MATH 1214 will have a knowledge base suitable for tackling Calculus I in an upcoming semester.

From the onset, it should be made clear that MATH 1214 does **not** satisfy the university's Area I Mathematics requirement and only students who plan to enroll in a future Calculus I class should take this course. Finally, to clear up confusion from past years, letter grades are assigned at the end of the course and do count towards one's grade point average.

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**STUDENT LEARNING OUTCOMES**     After successful completion of Functions, a Transylvania student will be able to:

- Demonstrate computational facility with the real number system including arithmetic of fractions, decimal expansions and square roots.
- Understand and be able to use the Cartesian coordinate system.
- Demonstrate computational and algebraic facility with polynomials, absolute value functions, power functions, exponential functions, trigonometric functions, and logarithmic functions.
- Compute and interpret the domains and ranges of the function types listed above.
- Sketch and interpret graphs of the function types listed above.
- Use algebraic and graphical means to solve equations and inequalities involving the function types listed above.

Throughout the term, a student's capability in each of these areas is assessed through midterm exams, a comprehensive final exam, homework sets, quizzes, and class participation.

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**CLASS**     The course meets Monday, Wednesday and Friday from 10:30 to 11:20 in BSC 108. Attendance at each and every class meeting is expected. If it is necessary for you to miss a class session, please inform Dr. Stufflebeam as soon as possible *prior* to your absence. Each unexcused absence beyond the second will result in a lowering of one's course grade by a full plus/minus letter grade. Arriving to class more than 10 minutes late will be considered an unexcused absence.

A flipped classroom will be employed throughout this course. Lectures will be videos; you are

responsible for watching a video before each class meeting. Homework will be assigned through the videos and should be worked (as completely as possible) before the next class meeting. At this next class meeting, we will study homework problems and additional problems from the material covered in the video. At the end of the class period, a quiz will be administered covering the homework.

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**TEXT** The required text for the course is *Precalculus, 3rd edition* (actually the  $[\pi]$  edition) by Carl Stitz and Jeff Zeager. It can be found - FOR FREE!!! - at [stitz-zeager.com](http://stitz-zeager.com). As an overwhelming majority of the assigned work in this course stems from this text, it is essential that you have continual access to a copy.

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**COURSE GRADING** The following percentages will be used in determining your final course grade:

Quizzes	—	25 %
Exams I - IV	—	15 % each
Final Exam	—	15 %

In calculating the quiz portion of your grade, the lowest *two* quiz scores will be dropped. During the course the following scale will be used; it may be expanded but will *not* be contracted: 100-93 A; 92-90 A-; 89-87 B+; 86-83 B; 82-80 B-; 79-77 C+; 76-73 C; 72-70 C-; 69-67 D+; 66-63 D; 62-60 D-; 59-0 F.

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**EXAMS** There will be four midterm examinations during the term. They are scheduled for Wednesday September 28, Monday October 24, Monday November 14, Monday December 5. If, for some reason, an exam needs to be rescheduled, notice will be given at least a week in advance. During pre-exam review sessions, the possible material to be covered on an exam will be discussed. Makeup exams will be given only in extraordinary circumstances. The Final Exam is cumulative and will be administered Friday December 16 from 12-2 PM in a location to be determined.

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**QUIZZES AND HOMEWORK** In order to learn mathematics, one must **do** mathematics. In an attempt to motivate your learning, homework assignments will be made on a daily basis at the end of each video lecture. Homework will not be turned in but it is expected that you have worked on it before each class period as we will discuss it during class. At the end of each (non-exam) period, a quiz over the homework assigned for that day will be administered. Your participation in class is welcome and expected. Though not an explicitly-defined part of the grade, your participation may be used to adjust the final quiz grade.

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**OFFICE HOURS** If my door is open, you are welcome to stop in and ask questions. Additionally, you may set up an appointment for a meeting. During the following times, I am guaranteed to be in (or around) my office:

Monday	— —	9 : 30 — 10 : 20, 1 : 30 — 3
Tuesday	— —	9 : 30 — 11 : 20
Wednesday	— —	9 : 30 — 10 : 20, 2 — 3 : 20
Friday	— —	9 : 30 — 10 : 20

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ACADEMIC INTEGRITY As detailed in the Transylvania University Course Catalog:

*“Honesty, trust, and personal responsibility are fundamental attributes of the University community. Academic dishonesty by a student will not be tolerated, as it threatens the foundation of an institution dedicated to the pursuit of knowledge. To maintain its credibility and reputation, and to equitably assign evaluations of scholastic and creative performance, Transylvania University is committed to maintaining a climate that upholds and values the highest standards of academic integrity.”*

It is expected that each of you will conduct your academic studies in a manner adhering to these guidelines. As mentioned above, you are encouraged to work with other students when studying and completing homework. It is expected that each individual submission reflect its author's understanding of the covered material. All exams and quizzes are individual endeavors and should be treated as such.

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CLASSROOM ATMOSPHERE Your participation is an integral part of the course. You are reminded, however, that there are other students in the class and non-mathematical off-topic discussions should be kept to a bare minimum. Drinks (water, coffee, etc.) are allowed in class, but all food items should be finished *before* lecture begins! iPhones, wireless headphones and all other feats of technology (save for the possibility of laptops or tablets - for the express purpose of taking notes) are to be turned *OFF* during class. Anyone using a cell phone or tablet, be it for talking, texting or surfing the net, during class will be asked to stop. A second such occurrence will result in that student being asked to leave class. Upshot: don't use your technology for non-educational reasons!!

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UP, DOWN, UP, DOWN, LEFT, RIGHT, LEFT, RIGHT, A, B, START The following is a set of recommendations to help you make your way successfully through the course. Mathematics is most definitely NOT a spectator sport; to perform to your potential and achieve success in Functions, you must make the effort and take the time to do and understand the homework problems and lecture examples. It is **imperative** that you watch the videos and attempt homework problems before class. As a rough estimate, one should expect to spend 3 hours studying outside of class for each class meeting (having your text open while watching *The Walking Dead* does not count!) Do not be afraid to make use of the other students in class as well as my availability. If questions arise or problems occur, please do not hesitate to contact me. Finally, please let me know of any circumstances which may affect your performance in the course.

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**TENTATIVE COURSE SCHEDULE** The following is the tentative daily schedule for MATH 1214-01. Changes to this schedule will be announced in class.

Monday	Wednesday	Friday
9/5 <b>No Class</b>	9/7 Intro	9/9 1.1-1.2
9/12 1.3-1.4	9/14 1.5-1.6	9/16 1.7
9/19 2.1	9/21 2.2	9/23 2.3
9/26 2.3	9/28 <b>EXAM I</b>	9/30 3.1
10/3 3.2	10/5 3.2-3.3	10/7 3.3
10/10 4.1	10/12 5.1	10/14 5.2
10/17 <b>NO CLASS</b>	10/19 5.2-5.3	10/21 5.3
10/24 <b>EXAM II</b>	10/26 6.1	10/28 6.1-6.2
10/31 6.2	11/2 6.3	11/4 6.3-6.4
11/7 6.4	11/9 8.1	11/11 8.1
11/14 <b>EXAM III</b>	11/16 10.1	11/18 10.2
11/21 10.2-10.3	11/23 <b>NO C</b>	11/25 <b>LASS</b>
11/28 10.3	11/30 10.5	12/2 10.5
12/5 <b>EXAM IV</b>	12/7 10.6	12/9 Review

The Final Exam is Friday December 16 from 12-2 PM in a location to be determined. For your reference, the drop date for the fall term is Monday September 19 and the withdrawal date is Friday October 28.

# MATH 1304-02 Calculus I

## Fall 2016

### Syllabus

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Dr. Ryan Stufflebeam  
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BSC 119                      233-8237

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**OVERVIEW**     Our primary goal in the course is to unearth the differential calculus, the beginnings of the integral calculus and the bridge between the two, the Fundamental Theorem of Calculus. Some of the topics to be encountered on our journey include: limits, continuity, derivatives and the corresponding differentiation formulas, applications of the derivative, trigonometric and transcendental functions and their derivatives, extrema, curve sketching and integration. A secondary goal of the course is to develop one's analytical and logical reasoning skills, useful traits for any discipline. A tertiary goal is to have fun!

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**STUDENT LEARNING OUTCOMES**     After successful completion of Calculus I, a Transylvania student will be able to:

- Compute limits involving algebraic, trigonometric, and certain transcendental functions and interpret the meaning of such computations.
- Determine the continuity of a function at a single point and on a set.
- Determine the differentiability of a function at a single point and on a set.
- Compute derivatives involving algebraic, trigonometric, and certain transcendental functions. Discover properties of the graphs of such functions from the computed derivatives.
- Understand graphical representations of functions and use such graphs to estimate first and second derivatives.
- Understand the Fundamental Theorem of Calculus and apply it in appropriate situations.
- Understand the difference between definite and indefinite integrals of algebraic, trigonometric, and certain transcendental functions.
- Set up, solve and explain problems modeled upon real world scenarios via calculus methods.

Throughout the term, a student's capability in each of these areas is assessed through midterm exams, a comprehensive final exam, homework sets, quizzes, and class participation.

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**CLASS**     The lecture portion of the course meets Monday, Wednesday and Friday from 11:30 to 12:20 in BSC 108. The laboratory portion of the class meets Tuesday from 1:30-2:45 in BSC 108. Attendance at each and every class meeting is expected. If it is necessary for you to miss a class or lab session, please inform Dr. Stufflebeam as soon as possible *prior* to your absence. Each unexcused absence beyond the third will result in a lowering of one's course grade by a full plus/minus letter grade. Arriving to class more than 10 minutes late will be considered an unexcused absence.

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**TEXT**     The required text for the course is James Stewart's *Essential Calculus: Early Transcendentals, 2nd Edition*. As an overwhelming majority of the assigned work in this course stems from this text, it is essential (ha!) that you have continual access to a copy.



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**COURSE GRADING**    The following percentages will be used in determining your final course grade:

Homework	—	15 %
Exams I - III	—	20 % each
Final Exam	—	25 %

In calculating the homework portion of your grade, the lowest *two* homework scores will be dropped. During the course the following scale will be used; it may be expanded but will *not* be contracted: 100-93 A; 92-90 A-; 89-87 B+; 86-83 B; 82-80 B-; 79-77 C+; 76-73 C; 72-70 C-; 69-67 D+; 66-63 D; 62-60 D-; 59-0 F.

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**EXAMS**    There will be three midterm examinations during the term. They are scheduled for  
Tuesday September 27,      Tuesday October 25,      Tuesday November 22

Note that these exams are scheduled during our weekly lab time. The location of these exams will be announced in class. If, for some reason, an exam needs to be rescheduled, notice will be given at least a week in advance. During pre-exam review sessions, the possible material to be covered on an exam will be discussed. Makeup exams will be given only in extraordinary circumstances. The Final Exam is cumulative and will be administered Thursday December 15 from 12 - 2 PM in a room to be determined.

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**HOMEWORK**    In order to learn mathematics, one must ***do*** mathematics. In an attempt to motivate your learning, homework assignments will be made on a daily basis. Generally, a homework set assigned during one class will be due at the beginning of the next class meeting. ***LATE homework will NOT be accepted.*** You are encouraged to work with others on homework. However, the work you submit should accurately reflect *YOUR* understanding of that day's material (i.e. Don't Cheat!). A few guidelines for the submission of homework: use standard paper *not* torn from a spiral notebook, staple multiple pages, show all work and write legibly in a coherent, logical manner. Your participation in class is welcome and expected. Though not an explicitly-defined part of the grade, your participation may be used to adjust the final homework grade.

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**LABORATORY**    Though this period is designated a *laboratory* period, it is actually a weekly period that can be used for various events. Three of these periods will be used to administer exams. Several other sessions will involve lecture and group discussion. Other Tuesdays will be used to solve problems in groups. The nature of each lab session will be announced in class each week. Attendance at each of these sessions is mandatory.

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**OFFICE HOURS**    If my door is open, you are welcome to stop in and ask questions. Additionally, you may set up an appointment for a meeting. During the following times, I am guaranteed to be in (or around) my office:

Monday	— —	9 : 30 — 10 : 20, 1 : 30 — 3
Tuesday	— —	9 : 30 — 11 : 20
Wednesday	— —	9 : 30 — 10 : 20, 2 — 3 : 30
Friday	— —	9 : 30 — 10 : 20

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**ACADEMIC INTEGRITY** As detailed in the Transylvania University Course Catalog:

*“Honesty, trust, and personal responsibility are fundamental attributes of the University community. Academic dishonesty by a student will not be tolerated, as it threatens the foundation of an institution dedicated to the pursuit of knowledge. To maintain its credibility and reputation, and to equitably assign evaluations of scholastic and creative performance, Transylvania University is committed to maintaining a climate that upholds and values the highest standards of academic integrity.”*

It is expected that each of you will conduct your academic studies in a manner adhering to these guidelines. As mentioned above, you are encouraged to work with other students when studying and completing homework. Also, your instructor knows all about online homework ‘engines;’ it is expected that you do not rely on these sites. It is expected that each individual submission reflect its author’s understanding of the covered material. All exams are individual endeavors and should be treated as such.

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**CLASSROOM ATMOSPHERE** Your participation is an integral part of the course. You are reminded, however, that there are other students in the class and non-mathematical off-topic discussions should be kept to a bare minimum. Drinks (water, coffee, etc.) are allowed in class, but all food items should be finished *before* lecture begins! iPhones, wireless headphones and all other feats of technology (save for the possibility of laptops or tablets - for the express purpose of taking notes) are to be turned *OFF* during class. Anyone using a cell phone, be it for talking, texting, tweeting, instagramming etc. during class will be asked to stop. A second such occurrence will result in that student being asked to leave class. Upshot: don’t use your cell phones!!

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**UP, DOWN, UP, DOWN, LEFT, RIGHT, LEFT, RIGHT, A, B, START** The following is a set of recommendations to help you make your way successfully through the course. Mathematics is most definitely NOT a spectator sport; to perform to your potential and achieve success in Calculus, you must make the effort and take the time to do and understand the homework problems and lecture examples. It is strongly advised that you use the provided lecture schedule to read the material to be covered in class ahead of time; minimally, one should at least browse the material prior to lecture. In any case, it is **imperative** that you read the text. Do not wait until the night before an assignment is due to begin it. As a rough estimate, one should expect to spend 3 hours studying outside of class for each class meeting (having your text open while binge-watching *Stranger Things* does not count!) Do not be afraid to make use of the other students in class as well as my availability. Also, on-campus tutoring is available starting the second week of class. This tutoring is free and takes place Sunday through Thursday 7-9 PM in the Education Resource Center in Room 007 - library basement. This is a walk-in service so please use it at your discretion. If questions arise or problems occur, please do not hesitate to contact me. Finally, please let me know of any circumstances which may affect your performance in the course.

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**TENTATIVE COURSE SCHEDULE** The following is the tentative daily schedule for MATH 1304-02. Changes to this schedule will be announced in class.

Monday	Tuesday	Wednesday	Friday
	9/6 Intro-1.1	9/7 1.1-1.3	9/9 1.2
9/12 <b>EXAM 0</b>	9/13 1.3	9/14 1.3-1.4	9/16 1.4
9/19 1.5	9/20 1.6	9/21 2.1	9/23 2.1-2.2
9/26 2.2 - Review	9/27 <b>EXAM I</b>	9/28 2.3	9/30 2.3-2.4
10/3 2.4	10/4 2.5	10/5 2.5-2.6	10/7 2.6
10/10 Lab	10/11 2.7	10/12 2.7	10/14 3.1
10/17 BREAK	10/18 BREAK	10/19 3.2	10/21 3.3
10/24 3.3-Review	10/25 <b>EXAM II</b>	10/26 3.5	10/28 3.5
10/31 3.7	11/1 3.7	11/2 4.1	11/4 4.3
11/7 4.3	11/8 4.4	11/9 Lab	11/11 4.5
11/14 4.5	11/15 4.7	11/16 5.1	11/18 5.1
11/21 Review	11/22 <b>EXAM III</b>	11/23 NO C	11/25 LASS
11/28 5.2	11/29 5.2-5.3	11/30 5.3	12/2 5.4
12/5 5.4	12/6 5.5	12/7 5.5-Lab	12/9 Review

The Final Exam is Thursday December 15 from 12 - 2 PM in a room to be determined. For your reference, the drop date for the fall term is Monday September 19 and the withdrawal date is Friday October 28.

# MATH 2144 Calculus III

## Fall 2014

### Syllabus

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Dr. Ryan Stuffelbeam  
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BSC 119                      233-8237

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**OVERVIEW**     This course is both a continuation and an extension of the first two courses in the Calculus series. Geometrically, instead of considering plain old points in the plane (pun intended), we will occupy our time with vectors in multi-dimensional space. Functionally, we will deal with functions of several variables whose outputs can be simple scalars or multi-dimensional vectors.

The main objective of Calculus III is to extend and/or adapt all of the concepts from single-variable calculus previously encountered to multi-variable functions. This will lead us into such topics as partial differentiation, multiple integrals and line integrals. Our goal is to cover as much material from Chapters 10, 11, 12 and 13 as possible.

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**CLASS**     The course meets Monday, Wednesday and Friday from 12:30 to 1:20 in CC 101 (Cowgill 101). Attendance at each and every class meeting is expected. If it is necessary for you to miss a class session, please inform Dr. Stuffelbeam as soon as possible *prior* to your absence. Each unexcused absence beyond the third will result in a lowering of one's course grade by a full plus/minus letter grade. Arriving to class more than 10 minutes late will be considered an unexcused absence.

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**TEXT**     The required text for the course is James Stewart's *Essential Calculus: Early Transcendentals, 2nd Edition*, the multi-variable option. As an overwhelming majority of the assigned work in this course stems from this text, it is essential (ha!) that you have continual access to a copy.

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**COURSE GRADING**     The following percentages will be used in determining your final course grade:

Homework	—	20 %
Exams I - III	—	20 % each
Final Exam	—	20 %

In calculating the homework portion of your grade, the lowest *two* homework scores will be dropped. During the course the following scale will be used; it may be expanded but will *not* be contracted: 100-93 A; 92-90 A-; 89-87 B+; 86-83 B; 82-80 B-; 79-77 C+; 76-73 C; 72-70 C-; 69-67 D+; 66-63 D; 62-60 D-; 59-0 F.

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**EXAMS**     There will be three midterm examinations during the term; each test will consist of an in-class portion as well as a take-home portion. They are scheduled for

Wednesday September 24,      Friday October 24,      Friday November 14

If, for some reason, an exam needs to be rescheduled, notice will be given at least a week in advance. During pre-exam review sessions, the possible material to be covered on an exam will be discussed. Makeup exams will be given only in extraordinary circumstances. The Final Exam is cumulative and will be administered Tuesday December 9 from 9 - 11 AM in CC 101.

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**HOMEWORK** In order to learn mathematics, one must **do** mathematics. In an attempt to motivate your learning, homework assignments will be made on a daily basis. Generally, a homework set assigned during one class will be due at the beginning of the next class meeting. **LATE** homework will **NOT** be accepted. You are encouraged to work with others on homework. However, the work you submit should accurately reflect *YOUR* understanding of that day's material (i.e. Don't Cheat!). A few guidelines for the submission of homework: use standard paper *not* torn from a spiral notebook, staple multiple pages, show all work and write legibly in a coherent, logical manner. Your participation in class is welcome and expected. Though not an explicitly-defined part of the grade, your participation may be used to adjust the final homework grade.

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**TECHNOLOGY** As we will be dealing with multi-dimensional (in particular 3D) objects, it is expected that students will become familiar with Maple - don't worry, we'll provide a solid introduction to Maple. Maple is a computer algebra system that allows users to manipulate - among other things - graphs of higher-dimensional objects. Maple 16 can be found on any computer in one of the on-campus computer labs.

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**OFFICE HOURS** If my door is open, you are welcome to stop in and ask questions. Additionally, you may set up an appointment for a meeting. During the following times, I am guaranteed to be in (or around) my office:

Monday	— —	8 : 30 — 9 : 20, 1 : 30 — 2 : 20
Tuesday	— —	9 : 30 — 11 : 20
Wednesday	— —	8 : 30 — 9 : 20, 11 : 30 — 12 : 20, 1 : 30 — 2 : 20
Friday	— —	8 : 30 — 9 : 30

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**ACADEMIC INTEGRITY** As detailed in the Transylvania University Course Catalog:  
*“Honesty, trust, and personal responsibility are fundamental attributes of the University community. Academic dishonesty by a student will not be tolerated, as it threatens the foundation of an institution dedicated to the pursuit of knowledge. To maintain its credibility and reputation, and to equitably assign evaluations of scholastic and creative performance, Transylvania University is committed to maintaining a climate that upholds and values the highest standards of academic integrity.”*

It is expected that each of you will conduct your academic studies in a manner adhering to these guidelines. As mentioned above, you are encouraged to work with other students when studying and completing homework. It is expected that each individual submission reflect its author's understanding of the covered material. All exams are individual endeavors and should be treated as such.

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**CLASSROOM ATMOSPHERE** Your participation is an integral part of the course. You are reminded, however, that there are other students in the class and non-mathematical off-topic discussions should be kept to a bare minimum. Drinks (water, coffee, etc.) are allowed in class, but all food items should be finished *before* lecture begins! Cell phones, iPods and all other feats of technology (save for the possibility of laptops or tablets - for the express purpose of taking notes or using Maple) are to be turned *OFF* during class. Anyone using a cell phone, be it for talking, texting or surfing the net, during class will be asked to stop. A second such occurrence will result in that student being asked to leave class. Upshot: don't use your cell phones!!

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UP, DOWN, UP, DOWN, LEFT, RIGHT, LEFT, RIGHT, A, B, START The following is a set of recommendations to help you make your way successfully through the course. Mathematics is most definitely NOT a spectator sport; to perform to your potential and achieve success in Calculus III, you must make the effort and take the time to do and understand the homework problems and lecture examples. It is strongly advised that you use the provided lecture schedule to read the material to be covered in class ahead of time; minimally, one should at least browse the material prior to lecture. In any case, it is **imperative** that you read the text. Do not wait until the night before an assignment is due to begin it. As a rough estimate, one should expect to spend at least 3.5 hours studying outside of class for each class meeting (having your text open while watching *The Big Bang Theory* does not count!) Do not be afraid to make use of the other students in class as well as my availability. If questions arise or problems occur, please do not hesitate to contact me. Finally, please let me know of any circumstances which may affect your performance in the course.

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**TENTATIVE COURSE SCHEDULE** The following is the tentative daily schedule for MATH 2144. Changes to this schedule will be announced in class.

Monday	Wednesday	Friday
	9/3 Intro-1.1	9/5 10.2
9/8 10.3	9/10 10.3-10.4	9/12 10.4
9/15 10.5	9/17 10.5	9/19 10.6
9/22 10.7	9/24 <b>EXAM I</b>	9/26 11.1
9/29 11.2	10/1 11.2-11.3	10/3 11.3
10/6 11.4	10/8 11.5	10/10 11.6
10/13 NO CLASS	10/15 11.6	10/17 11.7
10/20 11.8	10/22 11.8	10/24 <b>EXAM II</b>
10/27 12.1	10/29 12.2	10/31 12.3
11/3 12.5	11/5 12.6	11/7 12.7
11/10 13.1	11/12 13.2	11/14 <b>EXAM III</b>
11/17 13.2-13.3	11/19 13.3	11/21 13.4
11/24 13.5	11/26 NO CLASS	11/28 NO CLASS
12/1 13.7	12/3 13.8	12/5 13.9

The Final Exam is Tuesday December 9 from 9 - 11 AM in CC 101. For your reference, the drop date for the fall term is Monday September 22 and the withdrawal date is Friday October 31.

**MATH 2504 (= MATH 2<sup>3</sup> · 313)**

**The Mathematician's Toolkit**

**May Term 2016**

**Syllabus**

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**Overview** The Mathematician's Toolkit is a course designed to “bridge” the gap between the beginning calculus courses and the proof-oriented courses that make up the majority of a mathematics major. We will have three primary foci over the next four weeks: “doing” mathematics, “reading” mathematics, and “writing” mathematics. By focussing on the reading of mathematics, the writing of proofs, and the logic behind such endeavors, we hope to provide a solid introduction to the basics of higher mathematics. To exercise our mathematical muscles, we will spend our time studying logic, sets, number systems, equivalence relations, functions, and cardinalities. Two words of warning:

- (1) This course is intense and will keep you busy. The modes of thinking and the material developed in this class form the foundation of every upper-level mathematics course. Your future self will thank your present incarnation for your diligence this May Term. In other words, be prepared to think and work!
- (2) If you like mathematics, this course will be fun!

(Note: These two sentences are not contradictory.)

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**Learning Outcomes**

1. Students will gain an understanding of basic set theory, mathematical logic, and cardinality.
  2. Students will be able to understand the difference between a proof and an example.
  3. Students will gain experience in the process of ‘reading mathematics.’
  4. Students will gain experience in writing proofs and constructing counterexamples.
- 

**Course Objectives** There are two main objectives for this course. The first is meta-mathematical: to gain an appreciation of mathematics and what it takes to “do” mathematics. At a minimum, one should walk away from this class with the ability to read a mathematical text and precisely and accurately construct proofs and have an appreciation for the work ethic, persistence and patience needed to become a mathematician. The secondary goal is to become (intimately) acquainted with some proof techniques and basic mathematical concepts that are pervasive throughout mathematics. Proof techniques such as *reductio ad absurdum*, direct proof, proof by counterexample, equivalencies, contrapositive, etc. as well as mathematical topics such as truth tables and logic, quantifiers and mathematical statements, sets and set relations, cartesian products, equivalence relations, numbers systems, functions and cardinalities are to be studied.

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**Class** Class meets every day Monday-Friday from 1-3 PM in BSC 108. Attendance at each and every class meeting is expected. Please inform me as soon as possible if you will miss a class. Each unexcused absence after the first will result in the lowering of one's course grade by a full +/- . Arriving 15 or more minutes late to class is considered to be an unexcused absence.

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**Textbook** The primary textbook for the course is *Write Your Own Proofs* by Amy Babich and Laura Person. Lecture content and homework problems will primarily stem from this text. An auxiliary book for the course is Simon Singh's *Fermat's Enigma: The Epic Quest to Solve the World's Greatest Mathematical Problem*.

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**Grading** There will be three components in determining one's grade. These components are :

- HOMEWORK - 35%
- QUIZZES - 30%
- EXAM I - 15%
- EXAM II - 15%
- CLASS PARTICIPATION - 5%

The lowest quiz score will be dropped when calculating the quiz portion of your grade. The following scale will be employed throughout the next four weeks; it may be expanded but will not be contracted: 100-93 A, 92-90 A-, 89-87 B+, 86-83 B, 82-80 B-, 79-77 C+, 76-73 C, 72-70 C-, 69-67 D+, 66-63 D, 62-60 D-, 59-0 F.

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**Homework** Homework will be assigned on a near-daily basis. There will be 7 submission dates for completed homework. Those dates are:

	Monday May 2
Thursday May 5	Monday May 9
Thursday May 12	Tuesday May 17
Thursday May 20	Monday May 23

Submission dates for each set of selected problems will be clearly designated. Homework will be graded from two viewpoints: effort and correctness/precision. You are expected to do each and every problem assigned; a portion of each homework grade will reflect this. For each homework, you will be asked to select a given number of problems (usually 4-7) that you want graded for correctness/precision/mathematical grammar/etc. This will amount to a majority of the points on a given homework set.

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**Quizzes** A quiz will be administered at the start of 16 class periods (the exceptions are 4/27, 5/10, 5/11, and 5/24). Each quiz will cover material discussed in the previous class and, more than likely, material covered by the homework assigned the previous class.

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**Exams** There will be two exams. The entire class period will be dedicated to the exam. The exams are scheduled for

- TUESDAY MAY 10
- TUESDAY MAY 24



Topics to be covered will be discussed in class.

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**Class Participation** There will be numerous opportunities for each class member to participate during class. From presenting problems at the board to discussing Singh's book to general class discussion,

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**Office Hours** My office hours during May Term are

Monday	10 : 30–12 : 30	Tuesday	9 : 30–11 : 30	Wednesday	11 : 30–1 : 00
	Thursday	10 : 00 – 12 : 30	Friday	10 : 30 – 12 : 30	

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**Academic Integrity** As detailed in the Transylvania University Course Catalog:

*“Honesty, trust, and personal responsibility are fundamental attributes of the University community. Academic dishonesty by a student will not be tolerated, as it threatens the foundation of an institution dedicated to the pursuit of knowledge. To maintain its credibility and reputation, and to equitably assign evaluations of scholastic and creative performance, Transylvania University is committed to maintaining a climate that upholds and values the highest standards of academic integrity.”*

It is expected that each of you will conduct your academic studies in a manner adhering to these guidelines. As mentioned above, you are encouraged to work with other students when studying and completing homework. It is expected that each individual submission reflect its author's understanding of the covered material. All exams and quizzes are individual endeavors and should be treated as such.

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**Course Schedule** To help achieve the above objectives, the following is the tentative (and naively ambitious) course schedule for the course. Any alterations, amendments or plain old changes to this schedule will be detailed in class.

Day	Topics
Wednesday April 27	Introduction – Logic – The Joy of Sets
Thursday April 28	Quantifiers – <i>Discussion of Forewords</i> – Negations of Quantified Statements
Friday April 29	
Monday May 2	Even More Joy of Sets – Reductio ad Absurdum – <i>Chapter 1 of Singh</i>
Tuesday May 3	Direct Proofs – Two-Way Proofs – Counterexamples!!
Wednesday May 4	More Counterexamples – Cartesian Products – Relations
Thursday May 5	Equivalence Classes – <i>Chapter 2 of Singh</i>
Friday May 6	Indexed Sets – Partitions – Partial and Total Orders
Monday May 9	Basic Function Theory – <i>Chapter 3 of Singh</i>
Tuesday May 10	<b>EXAM I</b>
Wednesday May 11	Even More Function Theory – <i>Chapter 4 of Singh</i>
Thursday May 12	One-to-one – Onto – Inverse Functions
Friday May 13	Induction – <i>Chapter 5 of Singh</i>
Monday May 16	Even More Induction – Summation Notation – <i>Chapter 6 of Singh</i>
Tuesday May 17	Restrictions of Functions – The Pigeonhole Principle – Counting
Wednesday May 18	Cardinality – Power Sets
Thursday May 19	Infinite Sets – Countably Infinite Sets
Friday May 20	Uncountable Sets – Decimal Expansions of Real Numbers – <i>Chapter 7 and Epilogue of Singh</i>
Monday May 23	Cantor's Diagonal Argument
Tuesday May 24	<b>EXAM II</b>

# MATH 3224 Abstract Algebra

## Fall 2016

## Syllabus

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**Overview**    In part, mathematicians make a living by characterizing mathematical objects and classifying other structures that enjoy the same properties. During the course of this semester, we set out to classify and compare various number systems – the recognizable as well as the unfamiliar. Abstract algebra is the axiomatic study of number systems, their properties and their abstractions. For instance, considering the set of integers under the addition operation will lead to the general notion of a group. Similar examples will allow us to discuss rings, integral domains and fields.

Abstract algebra is a fundamental course for every math major. On one hand, its axiomatic treatment of algebraic systems allows one to begin a study of many advanced mathematical subjects (e.g. any topic of the form algebraic *arbitrary mathematical noun* or *arbitrary mathematical adjective* algebra). On the other, algebra provides, in a systematic and rigorous manner, the most general perspective of the different number systems we have worked with all of our lives. In this way, abstract algebra serves as a beginning and an end; though, hopefully for you, the next 13+ weeks will be the **start** of a fruitful relationship with this subject.

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**Student Learning Outcomes**    After successful completion of Abstract Algebra, a Transylvania student will be able to:

- Classify a mathematical structure as a group, ring, domain, field, or none of the above.
- Provide definitions and examples of subgroups, normal subgroups, and quotient groups.
- Construct rigorous proofs of propositions in group theory at large.
- Completely describe cyclic groups and finitely generated abelian groups.
- Provide definitions and examples of rings, domains, ideals and fields.
- Construct rigorous proofs of propositions in basic ring theory.
- Use and interpret the ideas of isomorphism and homomorphism for groups and rings.
- Understand and apply Lagrange's Theorem.

Throughout the term, a student's capability in each of these areas is assessed through midterm exams, a comprehensive final exam, homework sets, quizzes, and class participation.

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**Class**    Class meets every Monday, Wednesday and Friday from 8:30 to 9:20 in BSC 114. Your attendance is expected at each meeting. If you must miss class, please inform Dr. Stufflebeam as soon as possible *before* your absence. Each unexcused absence past the second will result in a lowering of one's course grade by a full plus/minus letter grade. Arriving at class more than 10 minutes late will be considered an unexcused absence.

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**Text**    The required text for the course is *A First Course in Abstract Algebra*, Seventh Edition, by John Fraleigh.

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**Course Grading**    The following percentages will be used in determining your final course grade:

Homework	—	15%
Exams I - III	—	15% each
Quizzes	—	20%
Final Exam	—	20%

In calculating the homework portion of your grade, the lowest score will be dropped. The analogous statement holds for your quiz grade. During the course the following scale will be used; it may be expanded but will *not* be contracted: 100-93 A; 92-90 A-; 89-87 B+; 86-83 B; 82-80 B-; 79-77 C+; 76-73 C; 72-70 C-; 69-67 D+; 66-63 D; 62-60 D-; 59-0 F.

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**Exams**    There will be **three** midterms during the course of the term. Each exam will consist of an in-class and take-home portion. The scheduled dates for these midterms are

Monday September 26,      Wednesday October 26,      Friday December 2

If, for some reason, an exam needs to be rescheduled, notice will be given at least a week in advance. During pre-exam review sessions, the possible material to be covered on an exam will be discussed. Makeup exams will be given only in extraordinary circumstances. The Final Exam is cumulative and will be administered Wednesday December 14 from 12 to 2 PM in BSC 114.

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**Homework**    In order to learn mathematics, one must **do** mathematics. Typically, homework will be assigned each Wednesday to be turned in a week later. **LATE** homework will **NOT** be accepted. You are encouraged to work with others on homework. However, the work you submit should accurately reflect *YOUR* understanding of that assignment's material (i.e. Don't Cheat!). A few guidelines for the submission of homework: use standard paper *not* torn from a spiral notebook, staple multiple pages, show all work and write legibly in a coherent, logical manner.

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**Quizzes**    Two hallmarks of a good mathematician are curiosity and precision. The curiosity needed to doggedly pursue a tough/interesting problem and the precision to accurately and technically present the fruits of one's pursuit cannot be overestimated. Homework assignments deal with the former trait; (somewhat) daily quizzes tackle the latter characteristic. On class days when a quiz is to be administered, the quiz will be available at 8:20 in BSC 114. You can begin at any time but the quiz must be turned in by 8:35 - when the lecture portion of class starts. Quiz material will consist of definitions, examples, statement of theorems and 'simple' proofs.

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**Office Hours**    If my door is open, you are welcome to stop in and ask questions. Additionally, you may set up an appointment for a meeting. During the following times, I am guaranteed to be in (or around) my office:

Monday	— —	9 : 30 — 10 : 30, 1 : 30 — 3
Tuesday	— —	9 : 30 — 11 : 20
Wednesday	— —	9 : 30 — 10 : 20, 2 — 3 : 30
Friday	— —	9 : 30 — 10 : 20

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**Academic Integrity** As detailed in the Transylvania University Course Catalog:

*“Honesty, trust, and personal responsibility are fundamental attributes of the University community. Academic dishonesty by a student will not be tolerated, as it threatens the foundation of an institution dedicated to the pursuit of knowledge. To maintain its credibility and reputation, and to equitably assign evaluations of scholastic and creative performance, Transylvania University is committed to maintaining a climate that upholds and values the highest standards of academic integrity.”*

It is expected that each of you will conduct your academic studies in a manner adhering to these guidelines. As mentioned above, you are encouraged to work with other students when studying and attempting homework. It is expected that each individual submission reflect its author's understanding of the covered material. All exams are individual endeavors and should be treated as such.

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**Classroom Atmosphere** Your participation is an integral part of the course; your comments and questions are welcomed and expected. Though not explicitly a part of your final grade, your level of participation may be used to positively influence your overall course grade. You are reminded, however, that there are other students in the class and non-mathematical off-topic discussions should be kept to a bare minimum. Drinks (water, coffee, etc.) are allowed in class, but all food items should be finished *before* lecture begins! iPhones, wireless headphones and all other feats of technology (save for the possibility of laptops/iPads being used to take notes) are to be turned *OFF* during class. Anyone using a cell phone, be it for talking, texting or surfing the net, during class will be asked to stop. A second such occurrence will result in that student being asked to leave class. Upshot: don't use your cell phones!!

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**Lather, Rinse, Repeat** The following is a set of recommendations to help you make your way successfully through the course. Mathematics is most definitely NOT a spectator sport; to perform to your potential and achieve success in the course, you must make the effort and take the time to do and understand the homework problems and lecture examples. *It is **EXPECTED** that you read the section(s) to be covered in class in lecture **BEFORE** coming to lecture.* The ability to successfully read mathematics is fundamental for any mathematics major/minor and is a trait that comes after much hard work. There may (will) be times where the reading may confuse you. Do not worry; as you get more exposure to the topics (in class and in re-reading the section), you will become more and more comfortable with them.

Do not wait until the night before an assignment is due to begin it. As a rough estimate, one should expect to spend 3–4 hours studying outside of class for each class meeting (Having your text open while watching highlights of European football matches does not count!) This class moves fast! I encourage you to engage in good study habits from the beginning of the course. Please take time to work on and think about each of the homework and presentation problems; many problems will require more than a minute's thought, so be persistent! Do not be afraid to make use of the other students in class as well as my availability. If questions arise or problems occur, please do not hesitate to contact me. Finally, please let me know of any circumstances which may affect your performance in the course.

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**Tentative Course Schedule** The following is the tentative (and ambitious!) daily schedule for MATH 3224. Changes to this schedule will be announced in class.

Monday	Wednesday	Friday
9/5 <b>NO CLASS</b>	9/7 Intro-1	9/9 2
9/12 2–3	9/14 3	9/16 4
9/19 4–5	9/21 5	9/23 6
9/26 <b>EXAM I</b>	9/28 6	9/30 8
10/3 8–9	10/5 9	10/7 10
10/10 10	10/12 11	10/14 11
10/17 <b>NO CLASS</b>	10/19 13	10/21 14
10/24 14	10/26 <b>EXAM II</b>	10/28 15
10/31 15	11/2 18	11/4 18–19
11/7 19	11/9 22	11/11 22
11/14 23	11/16 23	11/18 26
11/21 26	11/23 <b>NO CLASS</b>	11/25 <b>NO CLASS</b>
11/28 27	11/30 27	12/2 <b>EXAM III</b>
12/5 29	12/7 29	12/9 <b>REVIEW</b>

The Final Exam is Wednesday December 14 from 12 to 2 PM in BSC 114. For your reference, the drop date for the Fall Term is Monday September 19 and the withdrawal date is Friday October 28.

# MATH 3114 Higher Analysis

## Winter 2017

### Syllabus

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**OVERVIEW**     The goal of this course is to gain a deeper understanding of the real line and of functions of a single variable. In a sense, this course revisits many of the topics you encountered in the calculus sequence. However, the rigor and completeness of our approach will lead us to a deep and rich vein of mathematics. During this semester we will encounter limits, sequences and series, the standard topology on the real line, limits of functions, continuity, and differentiability.

The study of analysis entails learning many definitions; it is **IMPERATIVE** that you understand and familiarize yourself with the many definitions we encounter. Moreover, having standard examples and counterexamples for each definition will often come in handy. Also, it is **IMPERATIVE** that you read (and grapple with) the text. This is the surest way to gain entry into any field of mathematics. This course is going to challenge you; the beauty and power of analysis will make your efforts worth the while.

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**STUDENT LEARNING OUTCOMES**     After successful completion of Higher Analysis, a Transylvania student will be able to:

- Determine the basic topological properties of the real line and of subsets of the real line;
- Assess and prove the convergence of sequences and series;
- Determine the continuity and differentiability of functions defined on various subsets of the real line;
- Apply the Intermediate Value Theorem and the Mean Value Theorem in appropriate scenarios;
- Produce rigorous proofs of results and concrete counterexamples to false statements in the field of real analysis.

Throughout the term, a student's capability in each of these areas is assessed through midterm exams, class participation, weekly quizzes, and a comprehensive final exam.

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**CLASS**     This course meets Monday, Wednesday and Friday from 8:30 to 9:20 AM in BSC 114. Attendance at each and every class meeting is expected. If it is necessary for you to miss a class, please inform Dr. Stufflebeam as soon as possible *prior* to your absence. Each unexcused absence beyond the third will result in a lowering of one's course grade by a full plus/minus letter grade. Arriving to class more than 10 minutes late will be considered an unexcused absence. On the Fridays when a quiz is to be administered, the first 30 minutes of class will be devoted to the quiz, the final 20 will be devoted to learning new material.

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**TEXT**     The required text for the course is Stephen Abbott's *Understanding Analysis*, 2nd edition from Springer. As an overwhelming majority of the assigned work in this course stems from this text, it is necessary that you have continual access to a copy.

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**COURSE GRADING** The following percentages will be used in determining your final course grade:

Quizzes	—	20 %
Exams I - III	—	20 % each
Final Exam	—	20 %

In calculating the quiz portion of your grade, the lowest quiz score will be dropped. During the course the following scale will be used; it may be expanded but will *not* be contracted: 100-93 A; 92-90 A-; 89-87 B+; 86-83 B; 82-80 B-; 79-77 C+; 76-73 C; 72-70 C-; 69-67 D+; 66-63 D; 62-60 D-; 59-0 F.

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**EXAMS** There will be three midterm examinations during the term. They are scheduled for  
Friday February 3,      Friday February 24,      Friday March 31

If, for some reason, an exam needs to be rescheduled, notice will be given at least a week in advance. During pre-exam review sessions, the possible material to be covered on an exam will be discussed. Makeup exams will be given only in extraordinary circumstances. The Final Exam is cumulative and will be administered Wednesday April 19 from 9:00 - 11:00 AM in BSC 114.

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**QUIZZES AND HOMEWORK** In order to learn mathematics, one must **do** mathematics. In an attempt to motivate your learning, homework assignments will be made on a daily basis. You are encouraged to work with others on homework. However, you need to come to **YOUR OWN** understanding of that day's material as the quizzes are individual endeavors. On each Friday's quiz, you are required to show all work and write legibly in a coherent, logical manner. Your participation in class is welcome and expected. Though not an explicitly-defined part of the grade, your participation may be used to adjust the final quiz grade.

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**OFFICE HOURS** If my door is open, you are welcome to stop in and ask questions. Additionally, you may set up an appointment for a meeting. During the following times, I am guaranteed to be in (or around) my office:

Monday	—	10 : 30 — 11 : 20, 12 : 30 — 2 : 20
Tuesday	—	9 : 30 — 11 : 20
Wednesday	—	10 : 30 — 11 : 20, 1 : 30 — 2 : 20
Friday	—	10 : 30 — 11 : 20

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**ACADEMIC INTEGRITY** As detailed in the Transylvania University Course Catalog:

*“Honesty, trust, and personal responsibility are fundamental attributes of the University community. Academic dishonesty by a student will not be tolerated, as it threatens the foundation of an institution dedicated to the pursuit of knowledge. To maintain its credibility and reputation, and to equitably assign evaluations of scholastic and creative performance, Transylvania University is committed to maintaining a climate that upholds and values the highest standards of academic integrity.”*

It is expected that each of you will conduct your academic studies in a manner adhering to these guidelines. As mentioned above, you are encouraged to work with other students when studying and completing homework. It is expected that each individual submission reflect its author's understanding of the covered material. All exams and quizzes are individual endeavors and should be treated as such.

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**CLASSROOM ATMOSPHERE** Your participation is an integral part of the course. You are reminded, however, that there are other students in the class and non-mathematical off-topic discussions should be kept to a bare minimum. Drinks (water, coffee, etc.) are allowed in class, but all food items should be finished *before* lecture begins! Cell phones and all other feats of technology (save for the possibility of laptops or tablets - for the express purpose of taking notes) are to be turned *OFF* during class. Anyone using a cell phone, be it for talking, texting or surfing the net, during class will be asked to stop. A second such occurrence will result in that student being asked to leave class. Upshot: don't use your cell phones!!

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**LATHER, RINSE, REPEAT** The following is a set of recommendations to help you make your way successfully through the course. Mathematics is most definitely NOT a spectator sport; to perform to your potential and achieve success in any Analysis course, you must make the effort and take the time to do and understand the homework and quiz problems as well as lecture examples. It is strongly advised that you use the provided lecture schedule to read the material to be covered in class ahead of time; IT IS OF THE UTMOST IMPORTANCE THAT YOU READ THE TEXT!!! Do not wait until the night before a quiz or an exam to begin the associated homework. As a rough estimate, one should expect to spend 3.5-4 hours studying outside of class for each class meeting (having your text open while watching *Top Chef* does not count!) Do not be afraid to make use of the other students in class as well as my availability. If questions arise or problems occur, please do not hesitate to contact me. Finally, please let me know of any circumstances which may affect your performance in the course.

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**TENTATIVE COURSE SCHEDULE** The following is the tentative (and *extremely* ambitious) daily schedule for MATH 3114. Changes to this schedule will be announced in class.

Monday	Wednesday	Friday
1/9 Intro-1.1	1/11 1.2	1/13 <b>Quiz 1</b> , 1.3
1/16 <b>NO CLASS</b>	1/18 1.3	1/20 <b>Quiz 2</b> , 1.4
1/23 1.4	1/25 1.5	1/27 <b>Quiz 3</b> , 1.6
1/30 2.2	2/1 2.2	2/3 <b>EXAM I</b>
2/6 2.3	2/8 2.3-2.4	2/10 <b>Quiz 4</b> , 2.4
2/13 2.5	2/15 2.6	2/17 <b>Quiz 5</b> , 2.6
2/20 2.7	2/22 2.7	2/24 <b>EXAM II</b>
2/27 3.2	3/1 3.2	3/3 <b>Quiz 6</b> , 3.3
3/6 3.3	3/8 3.4	3/10 <b>Quiz 7</b> , 3.4
3/13 <b>SPRI</b>	3/15 <b>NG B</b>	3/17 <b>REAK</b>
3/20 4.2	3/22 4.2	3/24 <b>Quiz 8</b> , 4.3
3/27 4.3	3/29 4.4	3/31 <b>EXAM III</b>
4/3 4.4	4/5 4.5	4/7 <b>Quiz 9</b> , 4.5
4/10 5.2	4/12 5.2-5.3	4/14 <b>Quiz 10</b> , 5.3

The Final Exam is Wednesday April 19 from 9 - 11 AM in BSC 114. For your reference, the drop date for the winter term is Thursday January 26 and the withdrawal date for winter term is Friday March 3.



Books On Rigorous Educational Disciplines  
Suite 42, Reading Rainbow Plaza  
Nextlingo, YK 80504  
April 8, 2013

Students In Contemporary Mathematics (SICO), Ltd.  
BSC 108  
Lexington, KY 40508

To the Members of SICO,

It is my sincere pleasure to again write to you on behalf of the Science Department at Books On Rigorous Educational Disciplines (BORED). Not only was your resolution to Dr. Emubaffle's story "*Wikipedia Green and the Case of the Copious Colas*" a critical success (receiving favorable reviews from the *Kokomo Journal* and the Icelandic Review of Children's Books), its stellar sales have catapulted our company to the top of America's no-holds-barred books-with-minimal-readership market. Alas, as the market leader there is still work to be done.

Failing to learn a lesson from the previous two installments, our representatives again contracted Dr. Emubaffle to lay pen to paper and produce another Wikipedia Green caper. His latest effort is entitled "*Wikipedia Green and the Case of the Lousy Landlord*." This story details the attempt of Bugs Meany's father to become a landlord. Further verifying that the past is always prologue, Dr. Emubaffle has developed the plot of the story along with the relevant mathematics but has again been afflicted with a nasty case of writer's block. We again place the successful resolution of the story in your capable hands. Before being admitted into the ~~asylum~~ vacation spa, Dr. Emubaffle was heard muttering the words "amortization formula" and "equity." We believe these may play a role in your work.

We have again asked that you be split into groups, if you so desire, to increase our chances of receiving publishable material. The Science Department needs your versions of the resolution of the story by Noon on Wednesday April 17. Not wanting to totally abandon the project, Dr. Emubaffle has agreed to help you. However, he has put some limitations on his assistance. He will only discuss the details of the project when an entire group is present; this includes in his office and in e-mail messages. Also, he will be unavailable to discuss the story after 3:30 PM on Monday April 15. Good luck and we look forward to reading your work.

Sincerely,

P. I. O'Neer  
Manager, Science Department  
Books On Rigorous Educational Disciplines

P.S. The marketing department has found that catch phrases are now cliché and should be left out of the current installment. However, they would like the resolution to include a humorous exchange between Wiki and Bugs concerning Bugs' inability to do elementary mathematics.

## Wikipedia Green and the Case of the Lousy Landlord

"You know Wiki, the similarity is quite creepy," Sara said, craning her neck over the sofa to see what Wiki was up to.

"Huh?" Wiki replied absentmindedly staring at a plethora of strange symbols on the floor-to-ceiling blackboard in his bedroom.

"I was just saying that you and Sheldon from the *Big Bang Theory* might as well be the same person. I feel like I'm watching you 10 years in the future - except his T-shirts are much cooler than anything you could ever possibly pull off," Sara finished with a sarcastic smile.

Wiki, colored chalk in hand, turned to Sara and brusquely replied "What's wrong with my shirt?" Wiki then pointed at his double-pocket - hence, two pocket protectors! - tan Garanimals T-shirt.

Rolling her eyes, Sara said "Seriously?"

Wiki, not the best with nonverbal clues, changed topics. "My attempt to recreate Grigori Perelman's proof of the Poincare Conjecture has hit a roadblock. Want to run an errand with me?"

"I guess, anything has to be better than watching nerds on TV while my nerdy best friend does mathematics," Sara answered.

"Swell," Wiki replied. "I received a strange e-mail from Bugs Meany this morning. Through all the typos and nonsensical words, I think he and his father need my help with some sort of financial deal. I'm to meet them in 10 minutes at *The Grind* for coffee."

"Let's get moving then. The bus leaves from the corner anytime now," Sara said

As Wiki and Sara quickly made their way to the bus stop, Wiki turned to Sara and asked "This word *nerd*, what does it mean?"

Sara, suppressing a hearty laugh, said "Wiki, if you need to ask, you'll never know. C'mon, our bus is here."

After 5 minutes and an impromptu discourse by Wiki on why the wheels of the bus don't actually "go round and round," the bus arrived downtown and our dynamic duo disembarked and crossed the street to the coffee shop. Before entering, Sara turned to her companion and asked "Wiki, have you ever met Bugs' dad?"

"Come to think of it, I haven't," Wiki said.

As she opened the door and saw Bugs sitting next to a man wearing a shirt emblazoned with *My Other Shirt is at the Cleaners*, Sara muttered under her breath "This is going to be *verrry* interesting."

After buying a couple of double red eyes, Sara and Wiki made their way over to Bugs and his father. Out of character, Bugs rose, shook each visitor's hand and then introduced them to his father. Bugs' father stood up and said "Hi folks, name's Eeny Meany, but my friends call me Mo."

Sara immediately did a spit take into a conveniently-placed napkin. Oblivious to most humor and a bit nervous drinking coffee with his archenemy, Wiki cut to the chase. "So, gentlemen, what can we do for you?"

"Well," began Mr. Meany. "I watch a lot of HGTV and was astounded at how much money there is in the real estate market. I decided to buy a couple of houses that have been divided into apartments and become a landlord. "

"Gotcha so far," Wiki said as Sara nodded her head in agreement.

Mr. Meany continued "I ended up buying 2 places. The first place contains 4 apartments and the second

has 3. The 4-unit house cost \$600,000 which I financed through a 30-year mortgage with a nominal annual rate of 5% and monthly payments. The 3-unit house cost \$520,000 which I financed through a 30-year mortgage at 4.8% APR and monthly payments.”

“APR - All Playaz Rule!” bellowed Bugs.

Leaning over to Wiki, Sara whispered “OMG!”

“That’s *Annual Percentage Rate*, Bugs. Please continue Mr. Meany,” Wiki offered.

“Please, call me Mo. As I was saying,” Mr. Meany continued. “I thought my monthly payments were going to be \$500 and \$480 for each house. Thus, to make a huge profit, I charged each of the 4 tenants in the first unit \$500 a month and each of the 3 tenants in the second \$480 a month.”

“Excuse my brazenness, Mr. Meany – I mean Mo – but why did you think your monthly payments were going to be so low?” Wiki interrupted.

Mr. Meany sheepishly replied “I thought you just multiplied the rate number by a 100 to take it out of a percentage and decimalize it.”

Sara smirked and whispered “Decimalize?”

“Oh, Mo, no. To figure out your monthly mortgage payment, you need to use the amortiz-” began Wiki.

“Hammer Time! Dun na na na, da da, da da, Can’t Touch This!” Bugs yelled as he danced around the table. Unbelievably, he was wearing solid gold parachute pants. Sara, not wanting to be seen, cowered into the corner.

“Son!” bellowed Mr. Meany. “What, in all that is holy, are you doing?”

“Four-eyes mentioned Hammer Time, so I thought it was time to dance!” Bugs gleamed.

An unsettled Wiki said “I was going to say amortization formula, but your impoliteness cut me off. May I continue?”

“Yes,” answered a dejected Bugs.

Wiki cleared his throat. “As I was saying, you need to use a mathematical result by the name of the Amortization Formula to determine your monthly payment.” Wiki paused. Sara recognized the dorky look of computation in Wiki’s eyes. “By my quick calculations, each of your monthly mortgage payments would be in the neighborhood of \$3000, give or take a couple of hundred. Man, with the total income from each house being \$2000 and 1440, you’re getting fleeced.

Mr. Meany, with a sheepish look, answered “Wait, it gets worse. After 3 years of losing money on these properties, I found a local mortgage company, *Dewey, Cheatham and Howe*, to refinance both places.”

Sara nodded and said “Man, I see where this is headed. Continue.”

“Yes, well I used the 3 years’ worth of equity on each home to take on a new 15-year mortgage on each property. For the 4-apartment place, I took on a 15-year mortgage for the remaining amount owed at 3.2% nominal annual interest with monthly payments. For the 3-apartment place, I took on a 15-year mortgage for the remaining amount owed at 2.8% APR - ”

Bugs jumped up and yelled “All Playaz Rule!”

Sara and Wiki laughed while Mr. Meany muttered “Why me?” under his breath.

“Anyways,” Mr. Meany said, “after making my first monthly payment on each of these new loans, I was losing even more money!” Fighting back tears, Mr. Meany left the table and headed to the restroom.

Bugs glowered at Wiki and said “I know we’ve had our problems Dorky McDork, but I would owe you one if you can help out my old man. He can’t figure out why his monthly payments are so high. If you could explain how the mortgage companies computed his monthly payments and then help him out by suggesting how much he should charge per apartment in order to break even each month, I would be much obligated.”

Sara sighed and said “That would be obliged, doofus.”

“Oblige yourself, ” Bugs quickly retorted, hands turning into fists.

“Easy, you two,” Wiki said. ‘No problem, Bugs. But you have to promise to not bully me for at least four weeks. Okay?”

“I guess,” said a visibly depressed Bugs.

Wiki smiled, pulled a calculator, pen and paper from his ever-present fanny pack and said “I’ll start by calculating the monthly payment for each of the original 30-year mortgages and then determine how much your father was losing on each property during a single month. Then, I will calculate the equity Mo had in each house after 3 years. Next, I will compute the monthly payments on each of the 15-year mortgages and calculate the monthly loss on each property. Finally, I will suggest rental prices for the apartments in either house that will enable your father to make a profit! Shall we get started?”

Sara stared heavenward and sighed “Calgon, take me away!”

Already starting to concentrate, Wiki said “I’ll take that as a yes.”

*Editor’s Note:* This is where Dr. Emubaffle ended. You should finish the story with the correct solution to the various problems. Also, the rest of the story to be informative, hilarious and award-winning. Good luck!!!!

## Supporting Materials - Professional Activity

In 2011 and 2012, I co-organized MathFest contributed paper sessions on incorporating writing and editing into the mathematics curriculum. After reviewing abstract submissions, Martin Montgomery, my co-organizer, and I selected 33 presentations suitable for inclusion in our sessions. The success of our sessions led the editors of *PRIMUS (Problems, Resources, and Issues in Mathematics Undergraduate Studies)* to ask us to serve as editors for a special issue on writing and editing in undergraduate mathematics courses. The number and strength of submitted papers afforded us the opportunity to add a second special issue on the topic. Martin and I were in charge of the entire editorial process: soliciting papers, seeking out reviewers, writing reports to authors, and making final recommendations to the journal's primary editors. The two special issues, *PRIMUS Volume 24, Numbers 6 and 7*, are password-protected but can be found at: <http://www.tandfonline.com/toc/upri20/24/6> and <http://www.tandfonline.com/toc/upri20/24/7>

The next several pages contain the preprint *The Composite Two-Step*. Eric Kaper, a 2013 Transylvania graduate and current University of Kentucky mathematics graduate student, and I began work on this topic in the summer of 2012 and continued our work over the summers of 2013 and 2014. In early 2015, we achieved our final results and this paper reflects the fruit of our research. The paper has been submitted and is current under review. This work is an outcome of the program's *Senior Seminar*. April York, a 2012 Transylvania graduate, discussed a related paper for her *Senior Seminar* presentation. Eric showed great interest in furthering the paper's results. He and I then spent more than three years working on the project.

# The Composite Two-Step

Eric Kaper and Ryan Stuffelbeam

## Abstract

Let  $a$  be a nonzero digit and  $b \in \{1, 3, 7, 9\}$ . To a positive integer  $k$ , alternately append  $a$  on the left and  $b$  on the right. This process yields two infinite sequences of integers dependent upon which side the appending process begins. We are interested in values of  $k$  that result in either of the sequences being composed entirely of composites. In particular, we seek the smallest such  $k$  that is relatively prime to the greatest common divisor of  $a$  and  $b$ .

## 1 Introduction

In [3], L. Jones began an investigation of integers that produce only composite numbers when the same digit is repeatedly appended to the right. For each  $d \in \{1, 3, 7, 9\}$ , Jones found numbers coprime to  $d$  which generate composite sequences. The authors of [2] built on this by determining likely candidates for the smallest ‘seed’ in each of the 4 cases.

Our investigation into a question posed in [3] takes things a ‘step’ further - pun intended. We select two digits -  $a$  and  $b$  - and alternately concatenate, or append,  $a$  on the left and  $b$  on the right (or vice versa) to a positive integer  $k$ .

Given digits  $a$  and  $b$ , the positive integer  $k$  is a *right seed* (or simply a *seed*) if  $k$  is coprime to  $\gcd(a, b)$  and if each number in the sequence

$$R(k, a, b) = \{k, kb, akb, akbb, aakbb, aakbbb, aaakbbb, aaakbbbb, \dots\}$$

is composite. Call  $R(k, a, b)$  the *right two-step sequence* with *generator*  $k$  and *appending digits*  $a$  and  $b$ . Let  $[a, b]_R$  denote the set of all right seeds for the appending digits  $a$  and  $b$ .

On the flip side, for digits  $a$  and  $b$ , the positive integer  $k$  is a *left seed* (or simply a *seed*) if  $k$  is coprime to  $\gcd(a, b)$  and if the sequence

$$L(k, a, b) = \{k, ak, akb, aakb, aakbb, aaakbb, aaakbbb, aaaakbbb, \dots\}$$

consists entirely of composites. Call  $L(k, a, b)$  the *left two-step sequence* with *generator*  $k$  and *appending digits*  $a$  and  $b$ . Let  $[a, b]_L$  denote the set of all left seeds for the appending digits  $a$  and  $b$ .

In either case, the subscript denotes the side on which the appending begins. For future reference, we begin the indexing of a two-step sequence at 0. Thus,

the element  $s_i$  is the value obtained after  $i$  appending operations. Also, there is a need to differentiate between left and right seeds. We will see that  $5995 \in [1, 1]_L$  and  $2200 \in [1, 1]_R$ . However,  $59951$  and  $1122001$  are both primes making  $[1, 1]_L$  and  $[1, 1]_R$  distinct sets with neither a subset of the other.

As in [3], we restrict  $b$  to the set  $\{1, 3, 7, 9\}$ . If  $b$  is even or 5, every number coprime to  $\gcd(a, b)$  is in  $[a, b]_R$  while multiples of 2 or 5 coprime to  $\gcd(a, b)$  are in  $[a, b]_L$ . In either case, the sets are infinite and easily constructed. When the appending digits share a prime factor, the coprime requirement of a seed throws out the obvious generators.

For a nonzero digit  $a$  and any  $b$  in  $\{1, 3, 7, 9\}$ , the obvious questions are

1. Is  $[a, b]_R$  nonempty? If so, what is the smallest element of  $[a, b]_R$ ?
2. Is  $[a, b]_L$  nonempty? If so, what is the smallest element of  $[a, b]_L$ ?

The rest of this paper discusses these questions.

## 2 Coverings, Repunits and the Cover-12 Method

Our first approach relies heavily on coverings of the integers. The concept of a covering goes back to Erdős [1].

**Definition.** A *covering* of the integers is a system of congruences  $n \equiv c_i \pmod{m_i}$ ,  $1 \leq i \leq t$ , such that every integer satisfies at least one of the congruences.

The simplest nontrivial covering of the integers is  $n \equiv 0, 1 \pmod{2}$ . The definition does not require a partition of the integers; some numbers may satisfy two or more of a covering's congruences.

Given digits  $a$  and  $b$  and a generator  $k$ , one wants a systematic way to determine if each element of a two-step sequence is composite. Coverings provide a way to do so. Roughly speaking, one associates a prime to each congruence in a given covering. Then, if the construction of the covering and the association of primes is viable, the  $n$ th value in the sequence will be divisible by the prime associated to the congruence satisfied by  $n$ . In this manner, the composite nature of each number in the two-step sequence is guaranteed. However, it remains to determine appropriate coverings and an association of primes in each case.

In [4], Jones and White use this type of approach to show that the sets  $[a, b]_R$  and  $[a, b]_L$  are nonempty and, in fact, infinite. This settles the first issue in the questions posed above. However, their work focused on the existence of coverings and did not account for the actual values of the digits  $a$  and  $b$ . As such, the coverings constructed are universal and the seeds determined are in the trillions and beyond. The coverings in [4] use 31 distinct primes. In the following sections we develop a variant covering method that allows an association with no more than 7 primes. This method allows the discovery of seeds with 10 digits or less and, in a majority of cases, the seeds are less than 150000.

Before discussing a detailed example, we motivate our approach in a general fashion for the right two-step sequence. Let  $a$  be any digit,  $b \in \{1, 3, 7, 9\}$  and  $k$  a generator. If one begins listing the base-ten expansions of elements in  $R(k, a, b)$ , the appending of digits on *both* sides of  $k$  necessitates the fixing of a predetermined length for  $k$ , call this length  $l$ . Hence, for a given pair of appending digits  $a$  and  $b$ , many values of  $l$  may be considered before a seed is encountered.

Repunits, and their prime factors, play an important role in our next set of observations.

**Definition.** For  $m \geq 1$ , the *length- $m$  repunit* is

$$r_m = \frac{10^m - 1}{9} = \underbrace{11 \dots 11}_{m \text{ occurrences}}$$

Given a base-ten expansion of a member of  $R(k, a, b)$ , one may factor  $a$  and  $b$  from their appended portions. The result leaves repunits as factors on either side of the constructed number. (For example, in  $R(k, a, b)$ ,  $s_9$  is  $10^{5+l}a(1111) + 10^5k + b(11111)$ . We encourage the reader to similarly construct other elements of  $R(k, a, b)$  for future reference.) This natural occurrence of repunits proves vital in selecting prime factors to attach to coverings. In particular, we use repunits possessing a significant number of distinct prime factors relative to their length.

A glance at a listing of repunit prime factorizations (see, among others, [5]) reveals that  $r_6$  has 5 distinct prime factors: 3, 7, 11, 13, and 37 - making  $r_6$  an ideal candidate. For any prime factor  $p$  of  $r_6$ ,  $10^6$  is congruent to 1 and  $r_6$  is congruent to 0 modulo  $p$ . Armed with this information, consider the element  $s_{17}$  of  $R(k, a, b)$ . By definition  $s_{17} = a \cdot r_8 \cdot 10^{9+l} + 10^9k + b \cdot r_9$ . For a prime divisor  $p$  of  $r_6$ ,  $s_{17}$  is congruent to  $a \cdot r_2 \cdot 10^{3+l} + 10^3k + b \cdot r_3$  modulo  $p$  and this last term is simply  $s_5$  in  $R(k, a, b)$ .

This is not a coincidence. Every 12 appending operations appends an additional copy of  $b \cdot r_6$  to the right and a copy of  $a \cdot r_6$  to the left. Also, a factor of  $10^6$  appears on the base-ten summands involving both  $k$  and  $a$ . Thus, this two-step sequence member will be congruent to the two-step sequence member 12 places before it in  $R(k, a, b)$  modulo any prime divisor of  $r_6$ .

This narrows our task to constructing a covering for the first 12 nonnegative integers, call it a *cover-12*. Once a cover-12 is established and prime factors of  $r_6$  are associated to its congruences, any prime dividing  $s_i$  divides  $s_{12j+i}$  for any  $j$ . It should come as no surprise that the construction of a cover-12 is predicated upon solutions to congruences modulo each of the prime factors of  $r_6$ .

To begin, select appending digits  $a$  and  $b$  and a length  $l$  for the unknown generator  $k$ . List the first 12 elements of  $R(k, a, b)$  -  $s_0, s_1, \dots, s_{10}, s_{11}$ . Let  $p$  be a prime factor of  $r_6$ . For each  $i = 0, \dots, 11$ , determine the value of  $k$  making  $s_i$  divisible by  $p$ . Repeat this for each prime factor of  $r_6$ . We use Maple for these computations.

The result of these 60 calculations is a  $12 \times 5$  table where each column corresponds to a prime divisor of  $r_6$  and a number in the  $i$ th row indicates the



congruence class of the value of  $k$  that makes  $s_{i-1}$  divisible by the corresponding prime. The values in this table are used to determine if a cover-12 associated with just 5 (or fewer) primes is possible. For each viable cover-12, the Chinese Remainder Theorem is applied to the congruences involving  $k$  to find a solution modulo the involved primes. A solution is not enough; to be a seed, a solution must meet the designated length requirement.

### 3 A Cover-12 Example

Let's attempt to find an element of  $[7, 1]_R$  of length 4. For each prime  $p = 3, 7, 11, 13, 37$ , list the base-ten expansions of the first 12 elements of  $R(k, 7, 1)$ , set each equal to 0 modulo  $p$  and solve the resulting congruences for  $k$ . This produces the following table where  $n$  is the number of appendings modulo 12:

n	p=3	p=7	p=11	p=13	p=37
0	0	0	0	0	0
1	2	2	1	9	11
2	1	2	5	1	15
3	0	5	4	11	5
4	2	5	0	9	8
5	1	6	1	10	7
6	0	6	5	3	0
7	2	4	4	7	11
8	1	4	0	2	15
9	0	1	1	5	5
10	2	1	5	7	8
11	1	0	4	6	7

One cover-12, with associated congruences for  $k$ , is

$$\begin{array}{ll}
n \equiv 0 & (\text{mod } 3) \\
n \equiv 7, 8 & (\text{mod } 12) \\
n \equiv 2 & (\text{mod } 4) \\
n \equiv 1, 4 & (\text{mod } 12) \\
n \equiv 5 & (\text{mod } 6)
\end{array}
\qquad
\begin{array}{ll}
k \equiv 0 & (\text{mod } 3) \\
k \equiv 4 & (\text{mod } 7) \\
k \equiv 5 & (\text{mod } 11) \\
k \equiv 9 & (\text{mod } 13) \\
k \equiv 7 & (\text{mod } 37).
\end{array}$$

The Chinese Remainder Theorem gives the solution  $k = 49032$  which is unique modulo  $r_6$ . However, we are seeking a seed of length 4 so 49032 is not viable. Alas, all is not lost. Another covering, with associated congruences for

$k$ , is given by

$$\begin{array}{ll}
n \equiv 2 & (\text{mod } 3) & k \equiv 1 & (\text{mod } 3) \\
n \equiv 9, 10 & (\text{mod } 12) & k \equiv 1 & (\text{mod } 7) \\
n \equiv 3 & (\text{mod } 4) & k \equiv 4 & (\text{mod } 11) \\
n \equiv 1, 4 & (\text{mod } 12) & k \equiv 9 & (\text{mod } 13) \\
n \equiv 0 & (\text{mod } 6) & k \equiv 0 & (\text{mod } 37)
\end{array}$$

Applying the Chinese Remainder Theorem to this system yields  $k = 2479$ , a valid length-4 member of  $[7, 1]_R$ . Better yet, the covering explicitly details a prime factor of each element of  $R(2479, 7, 1)$ : a repeating cycle through 37, 13, 3, 11, 13, 3, 37, 11, 3, 7, 7, 3.

2479 is the smallest member of  $[7, 1]_R$  we found via the cover-12 method; for smaller lengths either no cover was found or the solution did not satisfy the fixed length requirement. The necessity to account for the length of the generator is an extra impediment inherent to this method.

Though this method uses many fewer prime factors than the approach in [4], it comes with a cost. The cover-12 method uses the values of the appending digits to create a table of congruences involving  $k$ . Using the table to find a cover-12, applying the Chinese Remainder Theorem to the resulting system and validating the length of the seed involves a bit of work. Moreover, one must scrutinize the table to find every possible cover-12. On top of all this, this entire process is undertaken for varying lengths of  $k$  - until an element of  $[a, b]_R$  is located. The upside is the discovery of small seeds.

We used the cover-12 method to find right seeds in 28 of the 36 sets  $[a, b]_R$  under consideration. The exceptions are  $[6, 3]_R$ ,  $[9, 3]_R$ ,  $[6, 7]_R$ ,  $[7, 7]_R$ ,  $[9, 7]_R$ ,  $[3, 9]_R$ ,  $[6, 9]_R$ , and  $[7, 9]_R$ .

## 4 Need More Primes? Look To a Larger Repunit

A cover-12 eluded us in each of these 8 exceptions. For the exceptional cases with  $\gcd(a, b) > 1$ , this is not surprising; the appending digits share a prime factor with  $r_6$ , removing a prime from consideration in the cover-12 method.

In light of this, we seek a repunit with more than 5 prime factors. As  $r_{12}$  is the smallest repunit possessing more prime factors than  $r_6$ , it is the logical choice.  $r_{12}$  has 7 prime factors: 3, 7, 11, 13, 37, 101, and 9901.

Arguments and calculations in the cover-12 method are naturally adjusted to use  $r_{12}$  and a cover-24. The resulting table of congruences for the generator has 24 rows and 7 columns. The rows correspond to the two-step sequence elements  $s_0, s_1, \dots, s_{22}, s_{23}$  while the columns correspond to the prime factors of  $r_{12}$ . Again, the crux is to inspect the table for each and every possible cover-24. Once a cover-24 is located, an application of the Chinese Remainder Theorem yields a possible seed, subject to the designated length requirement.

The cover-24 approach succeeds in finding seeds of length 10 in  $[6, 7]_R$ ,  $[7, 7]_R$ ,  $[9, 7]_R$ , and  $[7, 9]_R$ . These right seeds along with the right seeds

determined via the cover-12 method are listed below.

Small Right Seeds

Case	Seed	Case	Seed	Case	Seed
$[1, 1]_R$	2200	$[4, 3]_R$	54615	$[7, 7]_R$	1134835262
$[2, 1]_R$	938	$[5, 3]_R$	92595	$[8, 7]_R$	638
$[3, 1]_R$	2076	$[6, 3]_R$	?	$[9, 7]_R$	1490744143
$[4, 1]_R$	11895	$[7, 3]_R$	25970	$[1, 9]_R$	21053
$[5, 1]_R$	159	$[8, 3]_R$	8066	$[2, 9]_R$	4266
$[6, 1]_R$	142593	$[9, 3]_R$	?	$[3, 9]_R$	?
$[7, 1]_R$	2479	$[1, 7]_R$	6409	$[4, 9]_R$	12509
$[8, 1]_R$	1290	$[2, 7]_R$	447	$[5, 9]_R$	109857
$[9, 1]_R$	10571	$[3, 7]_R$	23162	$[6, 9]_R$	?
$[1, 3]_R$	2368	$[4, 7]_R$	84	$[7, 9]_R$	6287118099
$[2, 3]_R$	113938	$[5, 7]_R$	258	$[8, 9]_R$	32472
$[3, 3]_R$	10948	$[6, 7]_R$	1316005328	$[9, 9]_R$	14344

As required, each seed is composite. The Chinese Remainder Theorem provides a solution modulo  $r_6 = 111111$ . When  $l = 6$ , a cover-12 for  $R(k, 6, 1)$  produced a solution of 31482. Adding  $r_6$  to this value gives a valid length-6 seed 142593. A similar scenario plays out in the discovery of 113938 in  $[2, 3]_R$ .

## 5 What About the Left?

We turn our focus to small seeds in  $[a, b]_L$ . The cover-12 method is easily adjusted to suit the left two-step sequence  $L(k, a, b)$ . The primary change is in the form of the congruences involving  $k$  and this stems directly from beginning the appending process on the left instead of the right. That said, the process is entirely analogous to the cover-12 method for right two-step sequences.

This adjusted cover-12 method produces left seeds of length 6 or less in 28 cases, the same 28 as in the search for right seeds. That no cover-12 was found for any of the 8 outliers  $[6, 3]_L$ ,  $[9, 3]_L$ ,  $[6, 7]_L$ ,  $[7, 7]_L$ ,  $[9, 7]_L$ ,  $[3, 9]_L$ ,  $[6, 9]_L$ , and  $[7, 9]_L$  is not unexpected.

Based on our prior work, the next logical step is to use the 7 prime factors of  $r_{12}$  and a cover-24 approach to tackle the exceptional cases. As in the right two-step scenario, the cover-24 process succeeds in finding 10-digit left seeds in  $[6, 7]_L$ ,  $[7, 7]_L$ ,  $[9, 7]_L$ , and  $[7, 9]_L$ . We have the following results for left two-step sequences.

Small Left Seeds

Case	Seed	Case	Seed	Case	Seed
$[1, 1]_L$	5995	$[4, 3]_L$	22906	$[7, 7]_L$	2902678537
$[2, 1]_L$	1959	$[5, 3]_L$	148176	$[8, 7]_L$	3270
$[3, 1]_L$	16654	$[6, 3]_L$	?	$[9, 7]_L$	1600156945
$[4, 1]_L$	7294	$[7, 3]_L$	9845	$[1, 9]_L$	26466
$[5, 1]_L$	687	$[8, 3]_L$	8066	$[2, 9]_L$	4266
$[6, 1]_L$	98148	$[9, 3]_L$	?	$[3, 9]_L$	?
$[7, 1]_L$	24791	$[1, 7]_L$	22920	$[4, 9]_L$	103411
$[8, 1]_L$	6513	$[2, 7]_L$	1666	$[5, 9]_L$	24220
$[9, 1]_L$	22373	$[3, 7]_L$	1265	$[6, 9]_L$	?
$[1, 3]_L$	23683	$[4, 7]_L$	84	$[7, 9]_L$	1967190095
$[2, 3]_L$	25049	$[5, 7]_L$	1738	$[8, 9]_L$	102507
$[3, 3]_L$	10948	$[6, 7]_L$	4461572896	$[9, 9]_L$	3080

Each seed is indeed composite and, in the case of  $[5, 3]_L$ ,  $r_6$  was added to create a seed of the required length. Since  $L(2902678537, 7, 7)$  consists entirely of composite values, so does  $R(290267853, 7, 7)$ . 290267853 is smaller than the right seed in  $[7, 7]_R$  produced by the cover-24 method and we have a new small seed in  $[7, 7]_R$ .

The right and left seeds listed are the smallest we found using the cover-12 and cover-24 methods. As noted in [2] when appending a single digit on the right, it is difficult to explicitly demonstrate that a given seed is the smallest. The computational power needed in trying to show a number is *not* a seed is tremendous and, given the lack of knowledge concerning extremely large primes, tenuous due to the probabilistic nature of prime-checking programs. However, in [2], the seeds 37, 4070, 891, and 10175 for the digits  $d = 1, 3, 7$ , and 9 were deemed likeliest to claim the title of ‘the smallest.’ Our seeds are similar in magnitude and we believe they provide a worthwhile starting point in the search for the ‘smallest’ right and left seeds.

## 6 The Factorization Method

When  $a = 6, 9, b = 3$  or  $a = 3, 6, b = 9$ , no right or left seeds are found using either of our covering methods. This led us to consider another approach, one we call the factorization method. Though this new method fails in finding seeds in these exceptional cases, it does produce very small seeds in other instances.

Notice that the even-indexed terms are the same in  $R(k, a, b)$  and  $L(k, a, b)$ . Such a value has base-ten expansion

$$a \cdot r_m 10^{m+l} + k 10^m + b \cdot r_m = a \left( \frac{10^m - 1}{9} \right) 10^{m+l} + k 10^m + b \left( \frac{10^m - 1}{9} \right)$$

where  $2m$  is the index and  $l$  is the fixed length of the generator  $k$ . The question motivating our new approach is: Can we factor the even-indexed terms of a two-step sequence?

Multiply the final expression above by 9 and let  $K = 9 \cdot k$ . Also, since the index varies, replace  $10^m$  by the variable  $x$ . The expression transforms into

$$a \cdot 10^l \cdot x^2 + [K + b - a \cdot 10^l]x - b,$$

a quadratic in  $x$ . Suppose this quadratic factors into a product of linear factors  $(Ax + B)(Cx + D)$  for unknown integer values  $A, B, C$ , and  $D$ . Then, we must have

$$AC = a \cdot 10^l \tag{1}$$

$$BD = -b \tag{2}$$

$$AD + BC = K + b - a \cdot 10^l \tag{3}$$

From (3), we have

$$k = \frac{1}{9}(AD + BC + a \cdot 10^l - b).$$

For appending digits  $a$  and  $b$ , suppose we find integral values of  $A, B, C$ , and  $D$  satisfying the above equations. Notice that none of these values are zero. Hence, for  $m \geq 1$ , the involved linear factors are greater than 1 forcing the corresponding even-indexed terms to be composite. Ensuring the composite nature of the generator ( $m = 0$ ) is left to us.

As indicated by equations (1) and (2), for each pair of factors of  $a \cdot 10^l$  and each pair of factors of  $b$ , we compute  $k$  as expressed above and check if it is integral and composite of the correct length  $l$ . (We wrote Maple programs to perform the above.) For the resulting  $k$  to be a seed, we must examine the relevant odd-indexed terms.

We consider the example of  $L(k, 7, 1)$  to highlight the approach to the odd-indexed terms. When  $l = 1$  the above process gives  $k = 8$ ; each of the even-indexed terms of  $L(8, 7, 1)$  are composite. The odd-indexed terms of  $L(8, 7, 1)$  are the same as the even-indexed terms of  $R(78, 7, 1)$ . We apply the factorization method to the even-indexed terms of  $R(k, 7, 1)$  with  $l = 2$  in hopes that 78 is an output. It turns out that it is. Thus, all of the elements of  $L(8, 7, 1)$  are composite and  $8 \in [7, 1]_L$ .

Better yet, since the  $n$ th term of  $R(78, 7, 1)$  is the  $(n+1)$ st term of  $L(8, 7, 1)$ ,  $R(78, 7, 1)$  consists entirely of composite values. 78 is smaller than the seed 2479 discovered by the cover-12 method. Thus, the factorization method has indirectly yielded another new small seed.

In 14 cases, the factorization method directly produces seeds smaller than those discovered by our covering methods. In 11 of these cases, an appropriate consideration of the opposite-stepped sequence (as above with  $R(78, 7, 1)$ ) reveals a seed smaller than the one constructed by our covering methods. The one-seed-only situations are  $[5, 7]_R$ ,  $[5, 7]_L$ , and  $[8, 9]_L$ .

#### Smaller Seeds Produced by Factorization Method

Case	Seed	Case	Seed	Case	Seed
$[2, 1]_R$	221	$[5, 7]_R$	62	$[2, 3]_L$	198
$[4, 1]_R$	441	$[6, 7]_R$	667	$[4, 3]_L$	473
$[5, 1]_R$	56	$[8, 7]_R$	95	$[1, 7]_L$	11077
$[7, 1]_R$	78	$[2, 1]_L$	21	$[2, 7]_L$	187
$[8, 1]_R$	9	$[4, 1]_L$	4411	$[5, 7]_L$	72
$[2, 3]_R$	2198	$[5, 1]_L$	6	$[6, 7]_L$	6677
$[4, 3]_R$	4473	$[7, 1]_L$	8	$[8, 7]_L$	957
$[1, 7]_R$	1107	$[8, 1]_L$	91	$[8, 9]_L$	8099
$[2, 7]_R$	18				

One can check that the four seeds less than 20 produced by the factorization method are the smallest in their respective sets.

## 7 The Next Step

Our methods have discovered small seeds in 64 of the 72 cases. The results and omissions leave us with a number of questions.

1. In 4 cases -  $[8, 1]_R$ ,  $[2, 7]_R$ ,  $[5, 1]_L$ , and  $[7, 1]_L$ , the smallest seed is known. In the other 58 successful cases, are the known seeds the smallest seeds? If not, how does one produce the smallest seeds?

On a related note, the smallest seeds in  $[6, 7]_R$  and  $[6, 7]_L$  discovered using the cover-24 method were 10 digits long. The factorization method produced much smaller seeds in both cases. This leads us to believe in the cases where  $a = 7, b = 7$  or  $a = 7, b = 9$  there are seeds with fewer than 10 digits. Is this so?

2. We did not exhibit any *small* right or left seeds when  $a = 6, b = 3$  or  $a = 3, b = 9$ . (The methods of [4] produce seeds using 31 distinct primes.) A seed in any of these cases must be relatively prime to 3 effectively restricting the pool of possible candidates by a third. Can one find a method to produce a small seed in these exceptional cases?
3. In 4 cases,  $a = b = 3$ ;  $a = 8, b = 3$ ;  $a = 4, b = 7$ ; and  $a = 2, b = 9$ , the cover-12 method produces the same ‘smallest’ right and left seeds. Are these coincidences or a feature of the involved appending digits?

## References

- [1] P. Erdős, On integers of the form  $2^k + p$  and some related problems, *Summa Brasil. Math.* **2** (1950) 113-123.

- [2] J. Grantham, W. Jarnicki, J. Rickert, S. Wagon, Repeatedly appending any digit to generate composite numbers, *Amer. Math. Monthly* **121** (2014) 416-421.
- [3] L. Jones, When does appending the same digit repeatedly on the right of a positive integer generate a sequence of composite integers? *Amer. Math. Monthly* **118** (2011) 153-160.
- [4] L. Jones, D. White, Appending digits to generate an infinite sequence of composite numbers, *J. Integer Seq.* **14** (2011) Article 11.5.7.
- [5] M. Kamada, Factorizations of 11....11 (Repunit), available electronically at <http://stdkmd.com/nrr/repunit/>



## Recommendation for Ryan Stufflebeam

Ryan Stufflebeam is a wonderful teacher! Somehow he manages to be both brilliant and accessible to undergraduates. He possesses a charisma, wit and intellect that (in my mind) qualify him as a “gifted” teacher; yet his work ethic is so great that I am confident he would be a wonderful teacher even if he were innately awkward. During my five years as division chair, I regarded Ryan as the second hardest working member of the division, right behind his mentor, Dave Shannon. Upon Dave’s retirement last spring, Ryan immediately rose to fill his shoes as the anchor of the math program. He is simply an outstanding teacher, colleague and mathematician, and I consider him highly deserving of renewal of the Bingham Award.

I have formally visited Ryan’s classes five times, observing him teach in courses ranging from the lowest entry-level math course to Senior Seminar. Additionally, due to the great overlap in upper-level physics and math students, I also regularly interact informally with Ryan and our shared students in the hallways and offices of Brown Science Center. I am continually impressed by the great respect students exhibit for Ryan coupled with the ease with which they approach him with questions. Furthermore, he continually impresses me with the skill with which he deftly explains abstract concepts and concrete algorithmic processes. (I should mention that in he regularly fills boards with formulae and proofs in his classes and only rarely refers to his notes.)

In most classes, Ryan lectures at a board in a manner that at least outwardly appears rather traditional, but it is an incredibly interactive form of lecture. Students and professor are in a constant, directed conversation about the formulae being derived upon the board. There is a sense that he is in the trenches with his students, helping them slog through the mire of mathematics. Typically, Ryan provides an overview to a given proof, and then expertly asks leading questions until the students flesh out all of the details...and Ryan always thanks the students for their contributions.



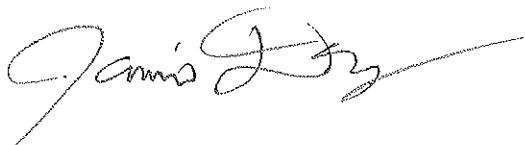
Ryan creates an environment where students also regularly ask questions of each other. Furthermore—at least in major classes—the student questions and student responses are usually of a complex nature, revealing both a deep involvement with the material and a deep trust and comfort with other students, which are signs of a true learning community.

In some classes, Ryan prepares a series of videos students watch outside of class. He effectively flips the class, so that rather than delivering content, he spends class time helping students one on one and in small groups to solve problems. Regardless of the style of a particular class, I have never seen students sitting passively in any of Ryan's classes. Instead, they are deeply engaged in the difficult work of learning very complex and abstract concepts.

Ryan is not perfect. He writes a bit like a medical doctor and tends to talk a little fast at times, but students do not seem to have difficulty interpreting his writing or his speech. Raised in Iowa farm country, Ryan has a few conversational habits that might distract grammarians, but which generally seem to make him more relatable to our students. I have rarely witnessed Ryan making a mistake in class, but on the one occasion I recall, he simply apologized and then turned the potentially awkward situation into a teaching opportunity by asking "What did I do wrong? Somebody tell me." After brief discussion among the students, someone soon did.

Charismatic, widely read, and curious, Ryan is an ideal teacher. He is confident without being arrogant, interactive without being pedantic, and enthusiastic without being phony. Although the learning curve in math is steep, Ryan manages to ask questions that are neither trivial nor impossible. He is clearly a gifted teacher, but he doesn't ride on his giftedness. Instead, he works hard. His courses are well designed, and his classes are well executed. Our students are lucky to have him as a teacher and Transylvania is fortunate to have him on our faculty. He has my strongest support for renewal of the Bingham Award.

Sincerely,

A handwritten signature in black ink, appearing to read "Jamie Day", with a long, sweeping horizontal line extending to the right.

Jamie Day  
Professor of Physics

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January 14, 2017

Members of the Bingham Renewal Committee,

It is my pleasure to write this letter recommending Dr. Ryan Stuffelbeam for renewal of his Bingham Fellowship. Dr. Stuffelbeam has been a friend and colleague for about 10 years. I have the highest respect for him as a teacher, a mathematician and a person. I know he is an innovative, effective teacher and I have learned a great deal from our conversations over the years. I know that Ryan's resume and personal statement have demonstrated his innovations and the breadth of his experiences. In this letter I would like to highlight just one course period in order to illustrate his incredibly effecting teaching style.

Ryan is great at building relationships. He is often seen chatting with students outside of his office about the sports they are in or some book they had read. He promotes our "Problem of the Week", our Putnam Exam along and many other activities. His students are very comfortable with him and find it easy to drop by his office and talk to him about anything. They trust that he cares about them and they work hard for him.

I have seen Ryan's abilities in the classroom often. In particular, I visited one of his Calculus 1 classes during the Fall semester of 2016. It is sometimes difficult for those outside of a field to appreciate all of the small, nuanced choices that go into a fantastic classroom experience. I would like to take the opportunity to break down what Ryan did during just this one 50-minute period to hopefully illuminate for you what a fantastic teacher Ryan is. While I will be speaking mainly about this one class, I can tell you from experience that the thought he puts into his classes and the techniques he uses are certainly not unique to this particular Calculus lecture but are indicative of all of his classes.

For the class that I visited Ryan was teaching on u-substitution. This is a foundational integration technique that most Calculus 1 students learn at the end of the semester. This can be a very dry lecture with a 'teach-by-example' approach. Ryan's approach was totally different as I will hopefully illustrate.

Research has shown that students learn best when new material is tied to old memories (either previous information learned in the class or every-day experiences). It has also been shown that learning is enhanced when material is brought forward at several different intervals during the semester. Ryan has certainly incorporated this research in his courses as can be illustrated in just this one period.

First Ryan reviewed a topic they learned the week before called the Fundamental Theorem of Calculus (FTC) that he called their “Old Friend”. He had previously proved this theorem and they had made use of it in subsequent homework. The current topic also makes use of this theorem. The FTC ties the ideas of derivation and integration together. He illustrated the idea with a boat going from “Derivative Land” to “Integration Island” and showed how the two parts of FTC go from derivation to integration and back again. The students loved playing into this analogy and were quick with similar analogies of their own.

In the same way, when he introduced u-substitution (the topic of the day), he wrote the formal definition on the board and then pointed out parts of the definition that related to previous material from the course. Instead of making the connections for them he asked them to think back by asking them “where did this come from? Do you recognize this?”. He was referring to the results from a derivative rule that they had learned weeks ago. U-substitution reverses this rule (using FTC) so by pointing out these similarities he was reviewing the previous derivative rule, reviewing FTC and illustrating the power of FTC while at the same time introducing u-substitution. He was also helping them to read formal mathematics by writing the formal definition and then helping them to ‘translate’ the language into something they are more familiar with.

After the formal definition he then talked informally about u-substitution (“it sweeps things under the rug”) in order to give the students a terrific concept basis before showing them the first example. He also built up some key phrases that he would use throughout the class to help students remember key parts of the process when they started to work on their own.

During the first example he specifically tied each part back to the theory. He had great board organization with labeling each example and solution as well as writing everything he said on the board. He had a great deal of enthusiasm and encouraged everyone to speak. He referred to students by name when they spoke and restated student questions so that everyone could hear and understand what was being asked.

He put the second example on the board and let the students **start** this second example in groups or alone as they preferred. Most classes would be terrified to attempt a problem after only seeing one example. However, Ryan had done such a good job with elaborating the concept on the first example that they had no problems with diving in to the second example. In fact, the whole class started right away which demonstrated to me that this was a very common practice in Ryan’s class. More importantly, this second example wasn’t simply the first example with different numbers. The second example had slight but important differences. He was not afraid to have them try an extension of the concept they had seen, allowing them the benefits of struggling just a bit and creating their own understanding.

Once they had worked for a bit, they started to notice the differences between his first example and this one. At this point he called their attention to the board again to talk about how to handle the missing constants. He again went back to previous material on derivatives to elaborate on how to handle constants in integration. He also discussed why the process would work well with constants but not with variables.

Each example he chose illustrated a different wrinkle but in each example he used the same phrases that tied back to the theory. He stressed issues that many students might forget in order to try and head off problems that students might encounter in their homework. In addition, Ryan printed off his own homework sheets with key reminders at the top of the page as an additional help.

After the two examples on the board he wrote two more examples and gave the class 5 minutes to work on them in groups. While they were working he walked around checking in with each group. He made a point of going back to students who had asked questions earlier to make sure that their questions were fully answered.

After the allotted 5 minutes he called the class back together to work both examples together. He started with the same phrases he used earlier “do I see someone and its derivative?”. He encouraged students to ‘think ahead’ as to what the answer might look like. When a student gave a wrong inverse trigonometry suggestion Ryan took the time to review the differences in the inverse trigonometry rules so that students would understand what would work in this situation.

At the end of the class he gave them one more example to work on. Everyone participated and no one packed up early. Ryan ended the class by reviewing that last example and discussing how the next class period would tie in to the material they just learned. By the end of the period the students seemed very comfortable with a very complicated topic.

Throughout this period Ryan demonstrated a clarity of focus in how each example led to the next, how he continually tied the theory to the thought process and how he foreshadowed the next class and even the next semester. He built awareness of how this topic fits in with the previous topics and with the theory of Calculus in general. I also want to point out how he promoted independence by allowing students to work on their own, showing them how to read mathematics and pointing out how the new material is similar to previous understanding.

I want to stress that all of this was demonstrated in just one 50-minute period but that these habits are very much ingrained in Ryan’s teaching philosophy. He does everything he can to help students learn the material in the best way possible. He is a remarkable teacher who is unafraid to try new techniques and he is always evaluating what he is doing to keep up with the latest research. He is very deserving of a renewal of his Bingham Award and I hope you give him strong consideration.

Sincerely,

Kim Jenkins  
Associate Professor of Mathematics